Digital Image - Lecture 07

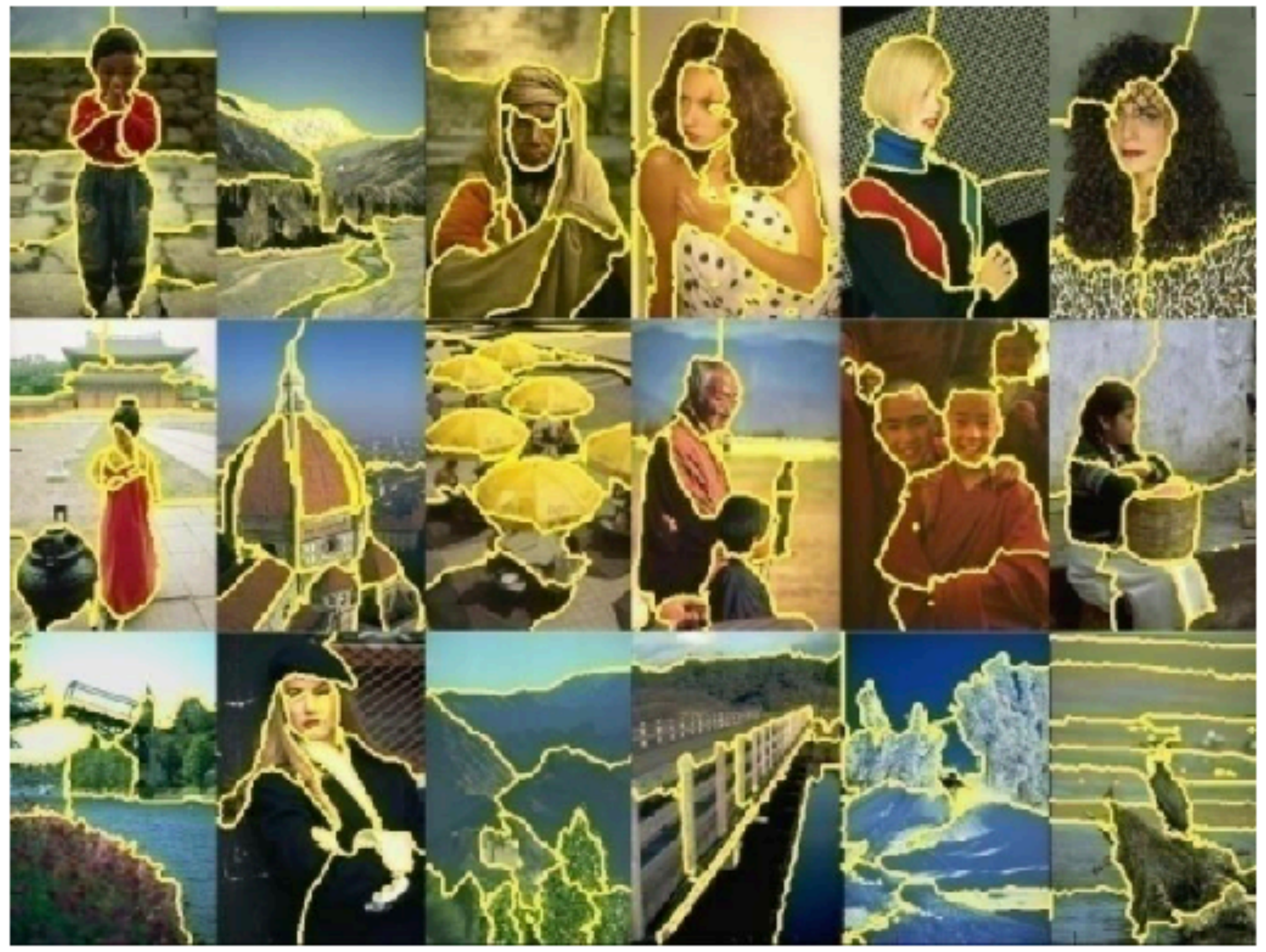
Image Segmentation
Part I

Torsten Sattler

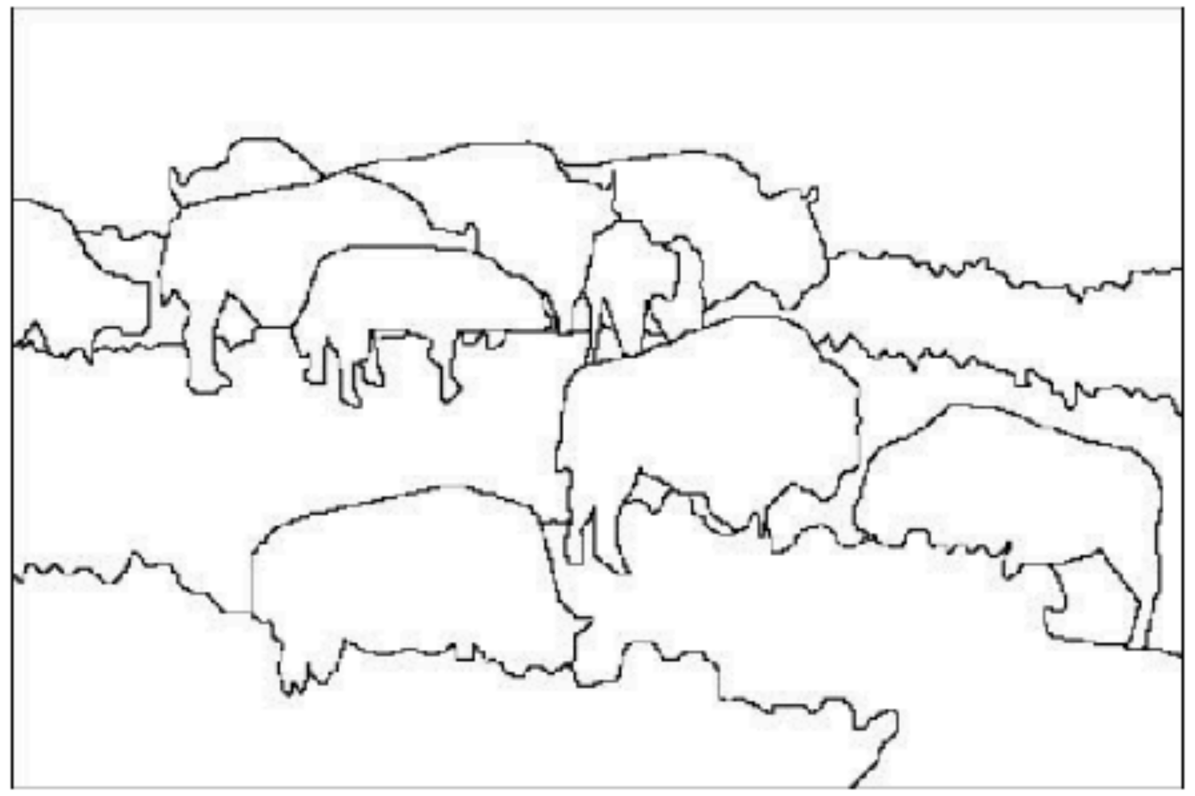
Czech Institute of Informatics, Robotics and Cybernetics
Czech Technical University in Prague

slides adapted from Václav Hlaváč and Bastian Leibe

What Is Image Segmentation?



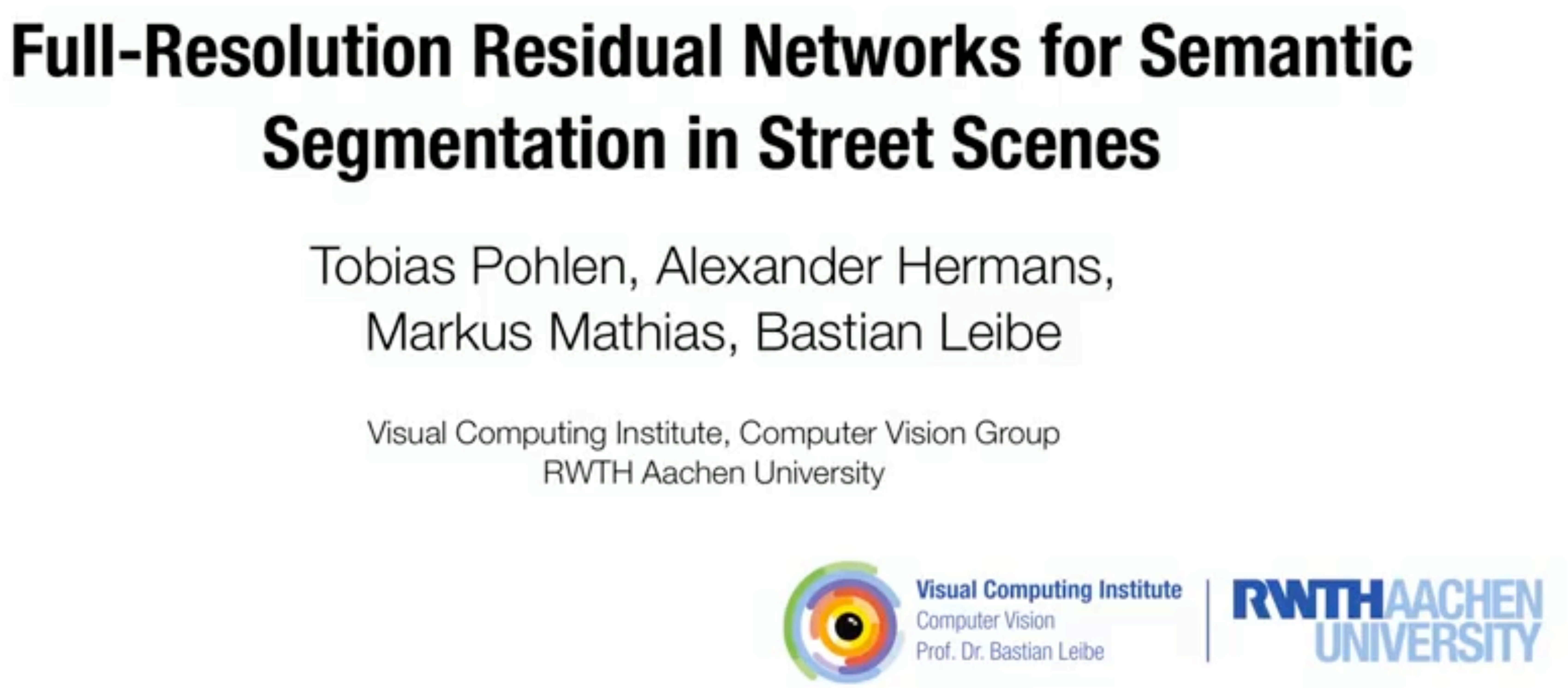
Image, courtesy Ondřej Drbohlav



Goal: **segment** image **into** (semantically) meaningful **regions**

slide credit: Václav Hlaváč, Bastian Leibe, Kristen Grauman, Svetlana Lazebnik

Example: Semantic Segmentation



[Pohlen, Hermans, Mathias, Leibe, Full-Resolution Residual Networks for Semantic Segmentation in Street Scenes, CVPR 2017] video link

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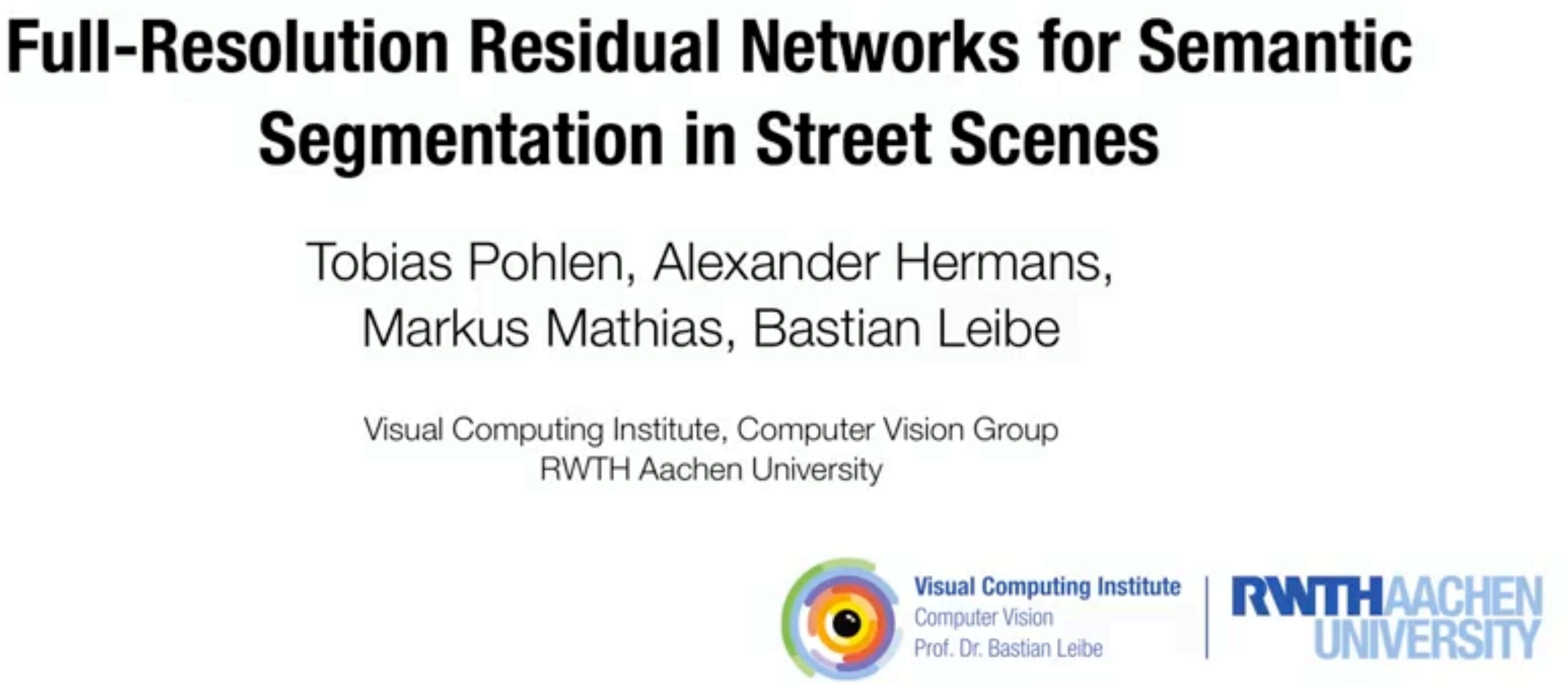

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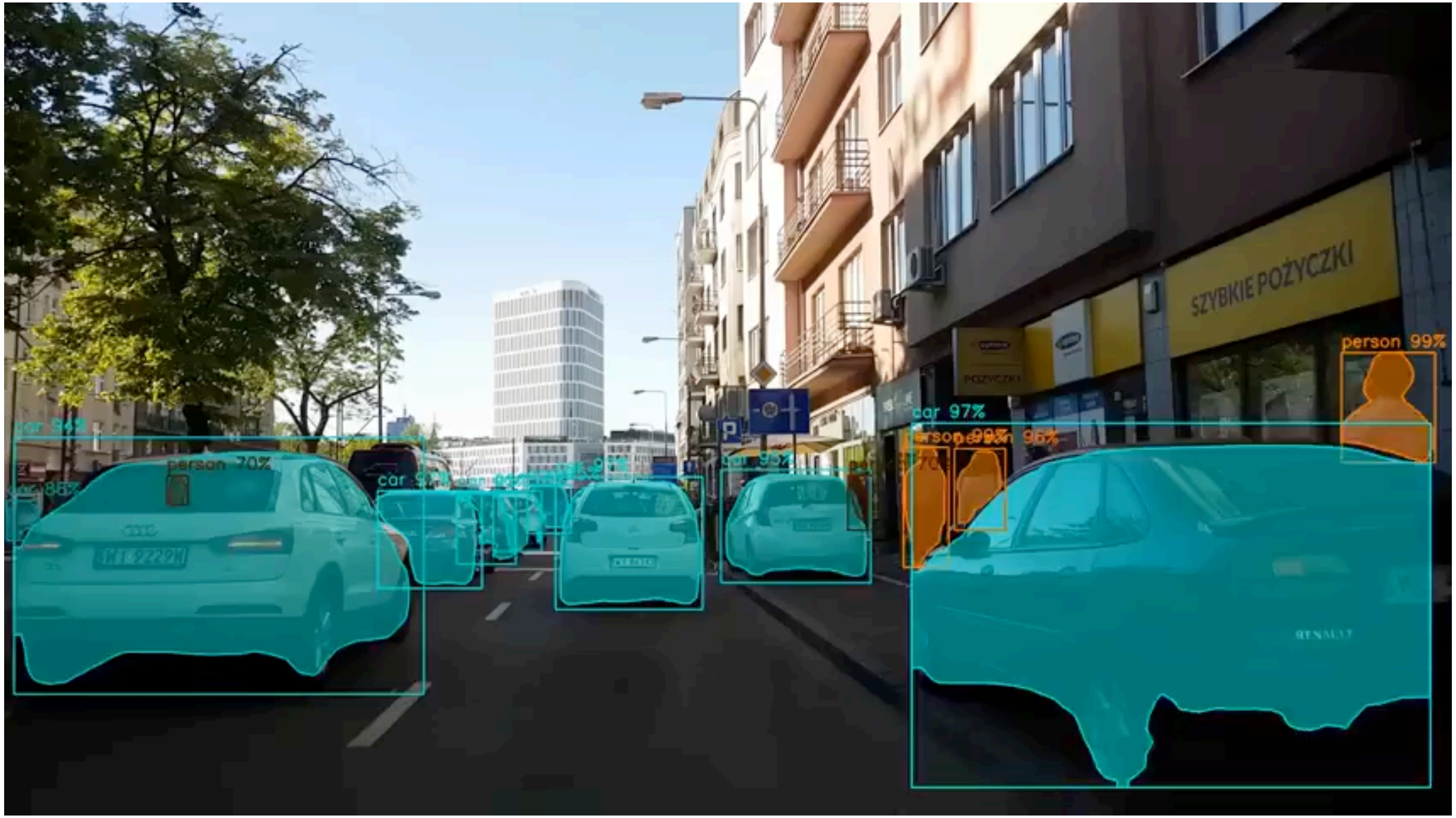
Example: Instance-Level Segmentation



video link

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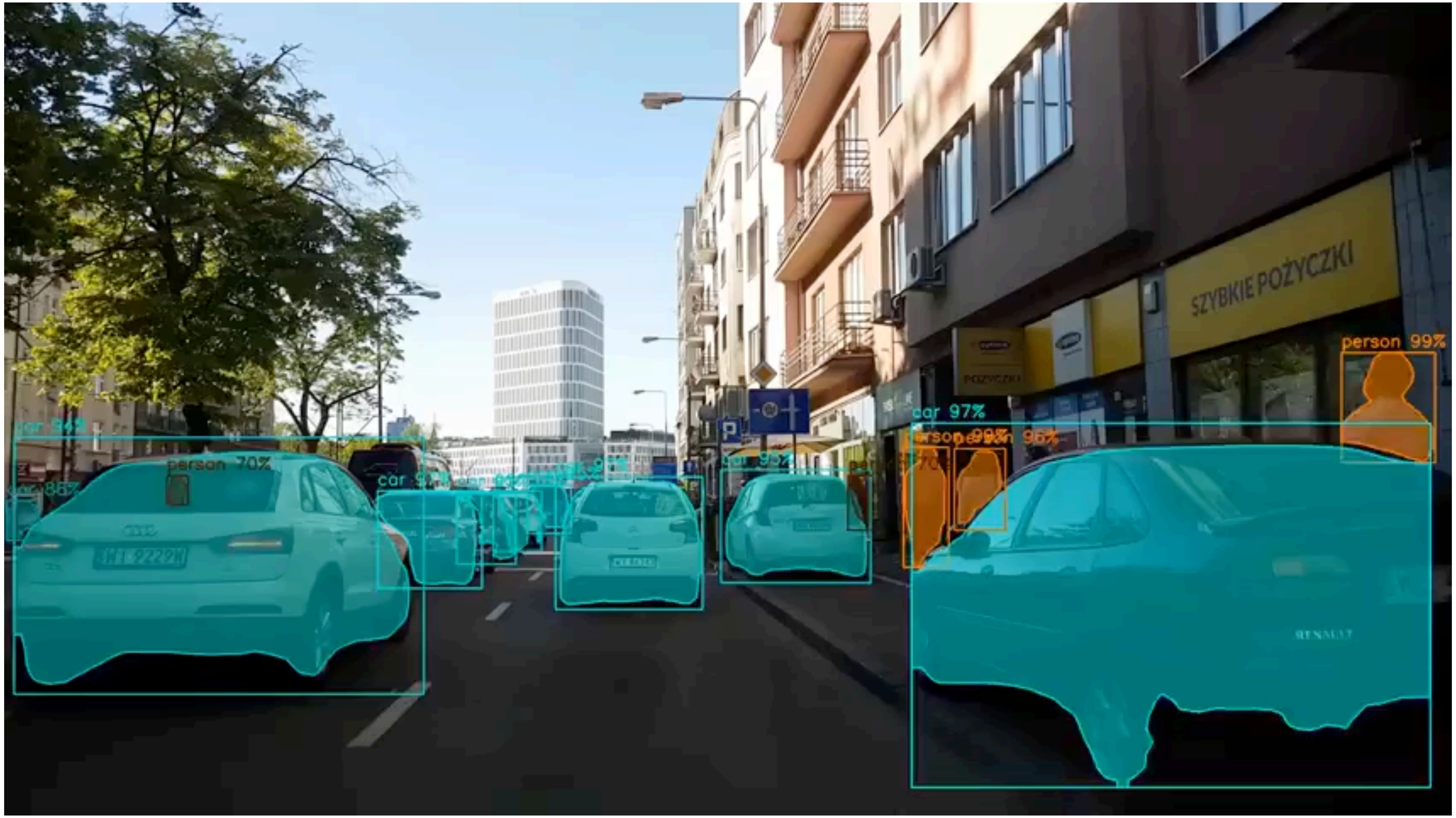
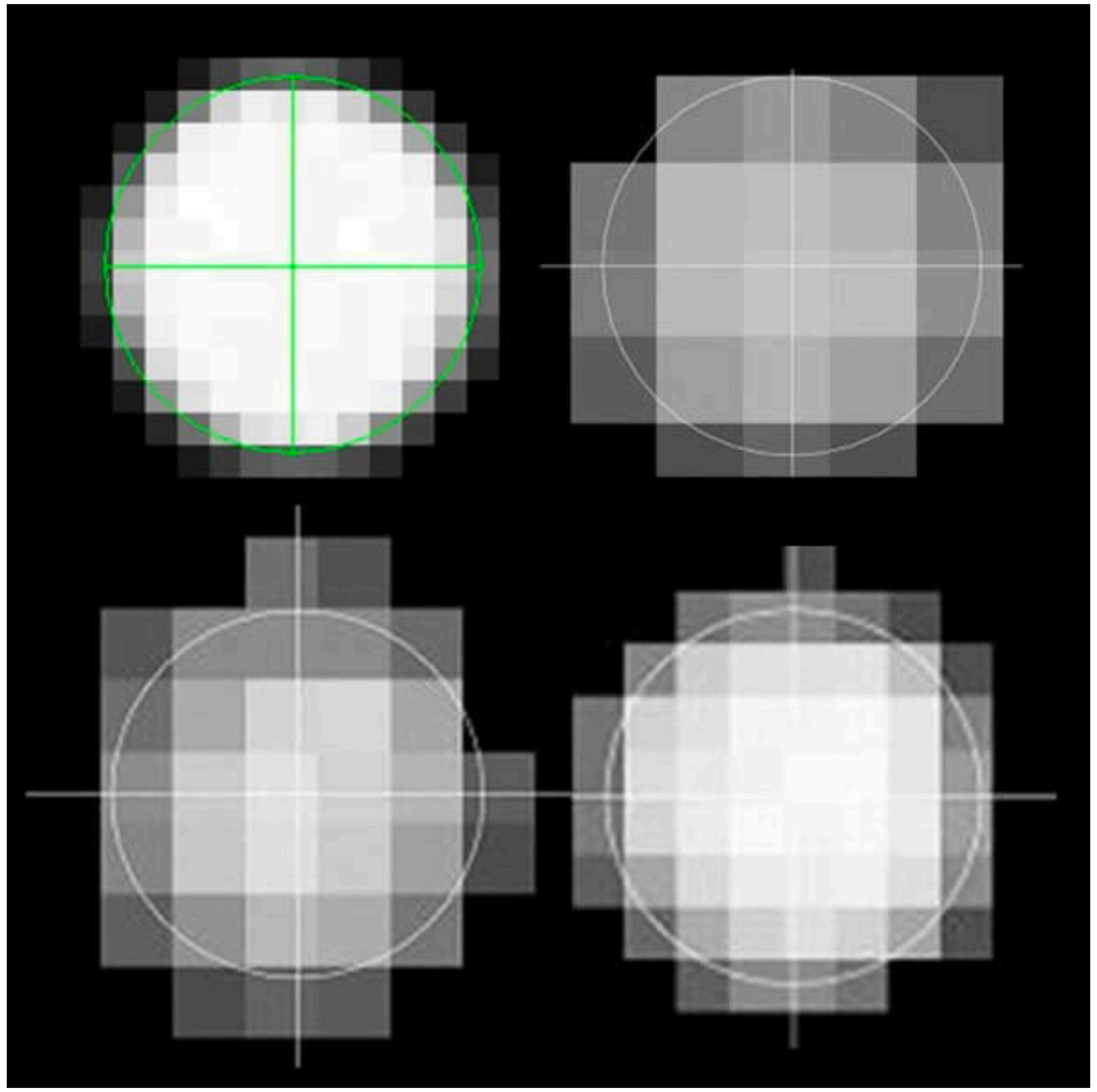


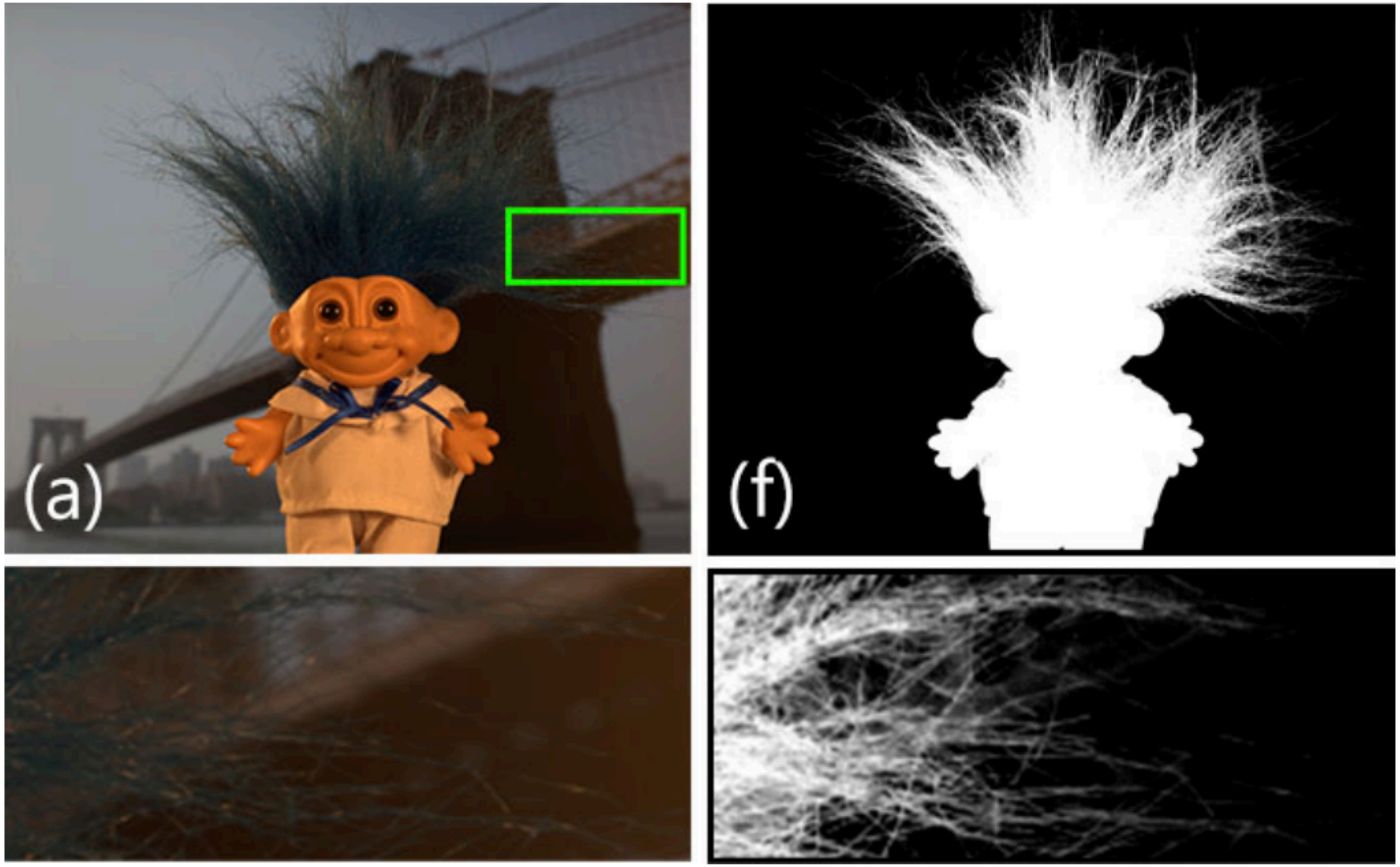
video link

Example: Motion Capture



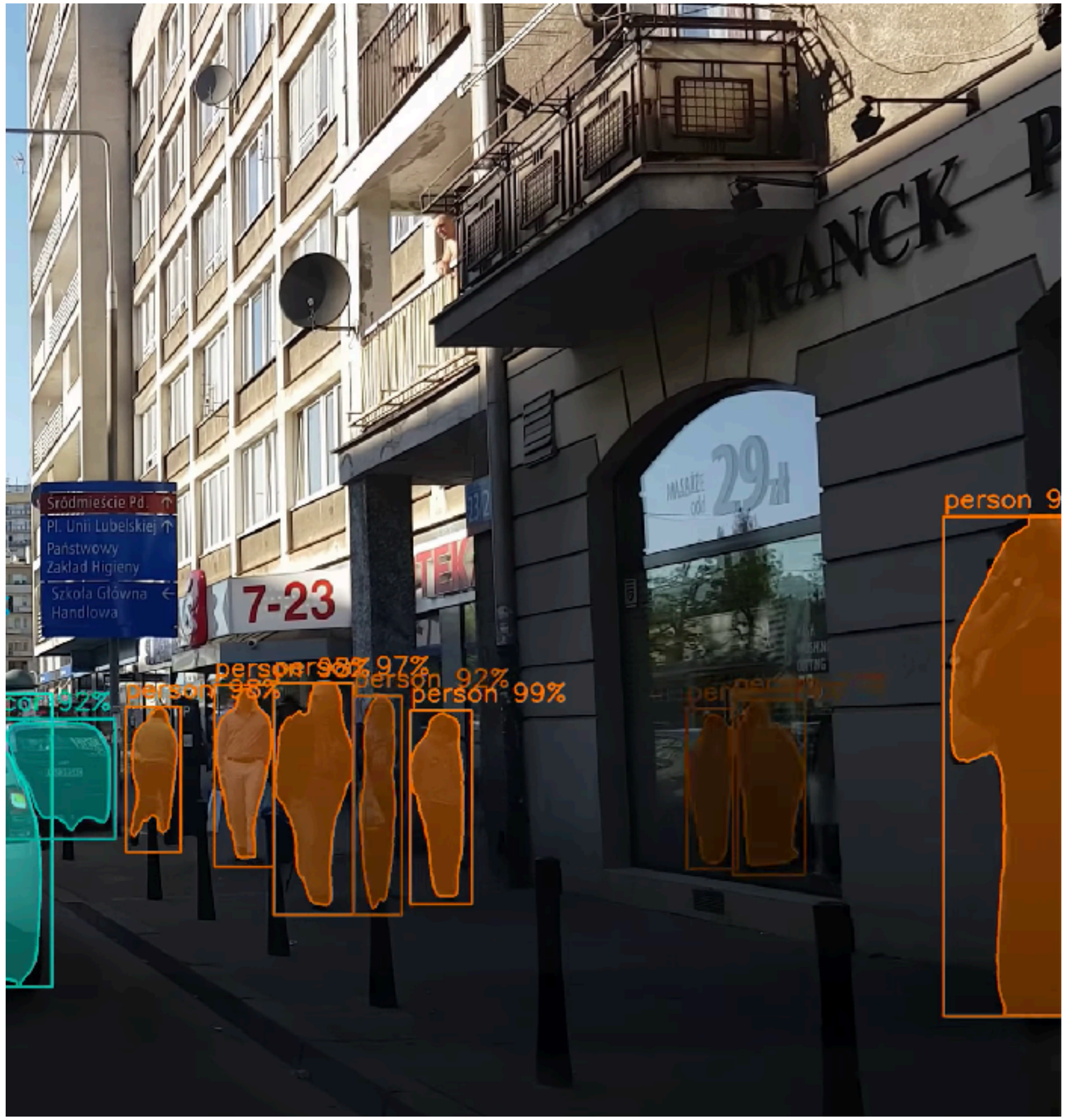
images taken from Vicon website

Example: Foreground Background Segmentation



[Yağız Aksoy, Tunç Ozan Aydın, Marc Pollefeys, Designing Effective Inter-Pixel Information Flow for Natural Image Matting, CVPR 2017]

Image Segmentation Can Be Hard



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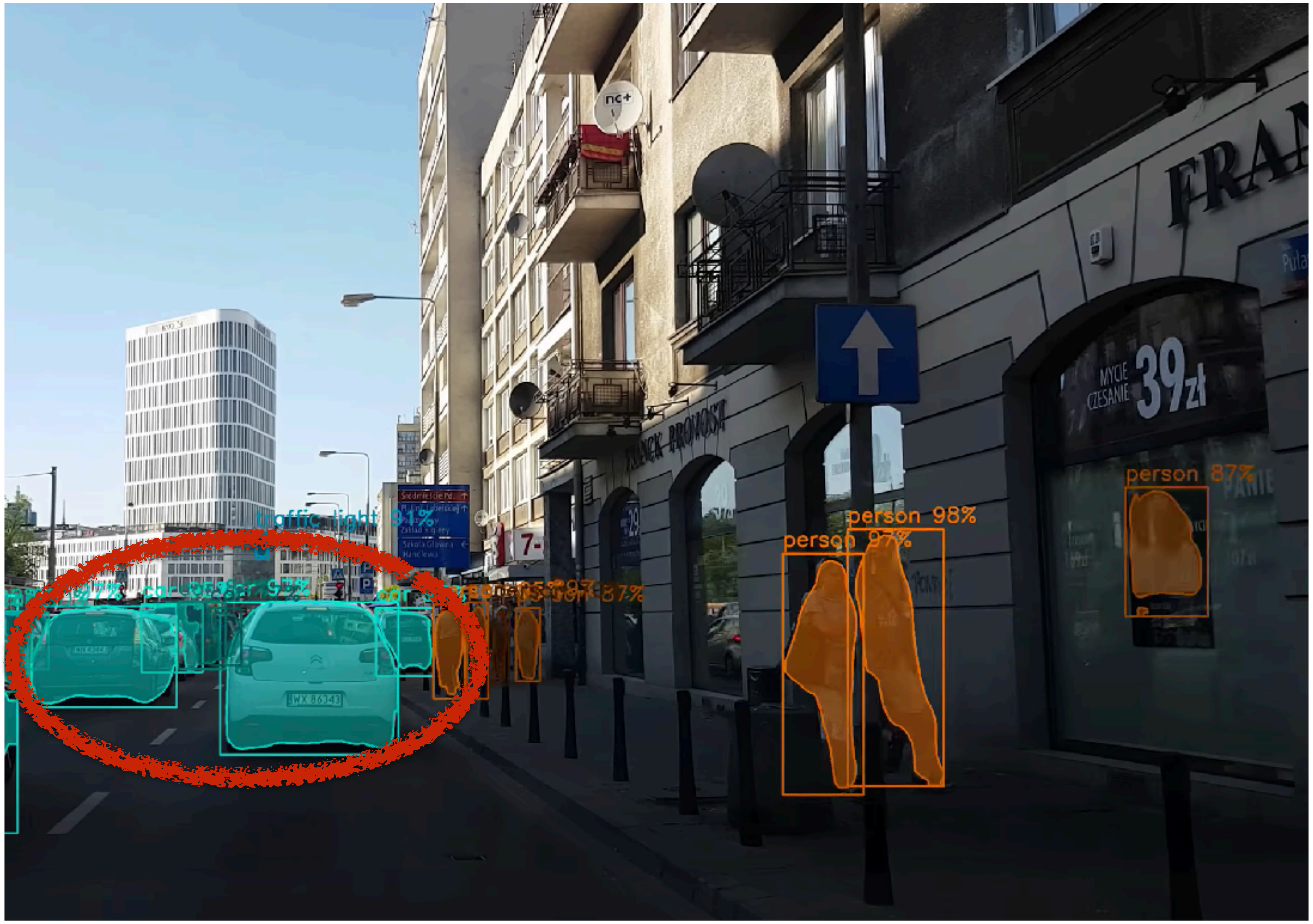

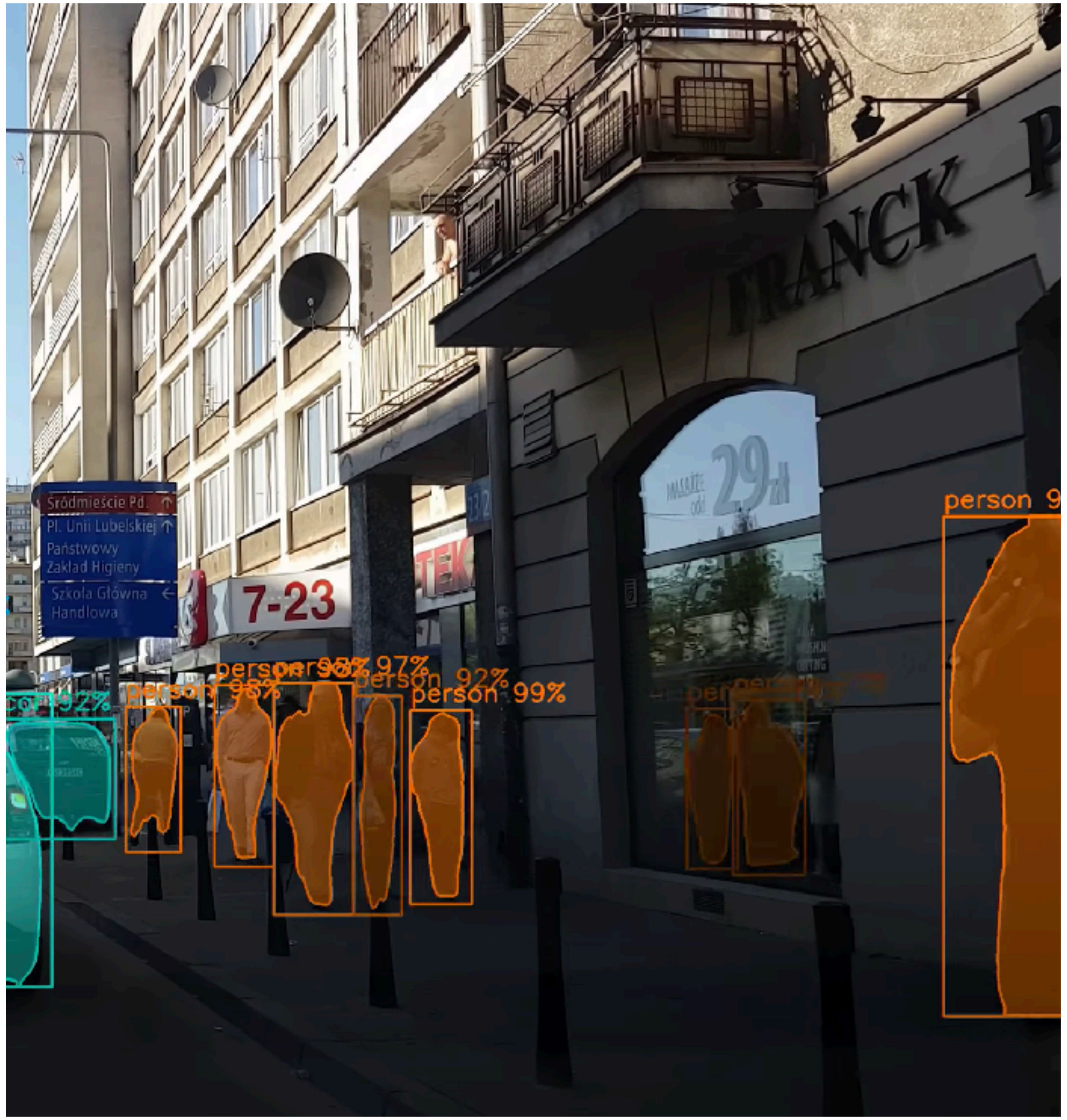

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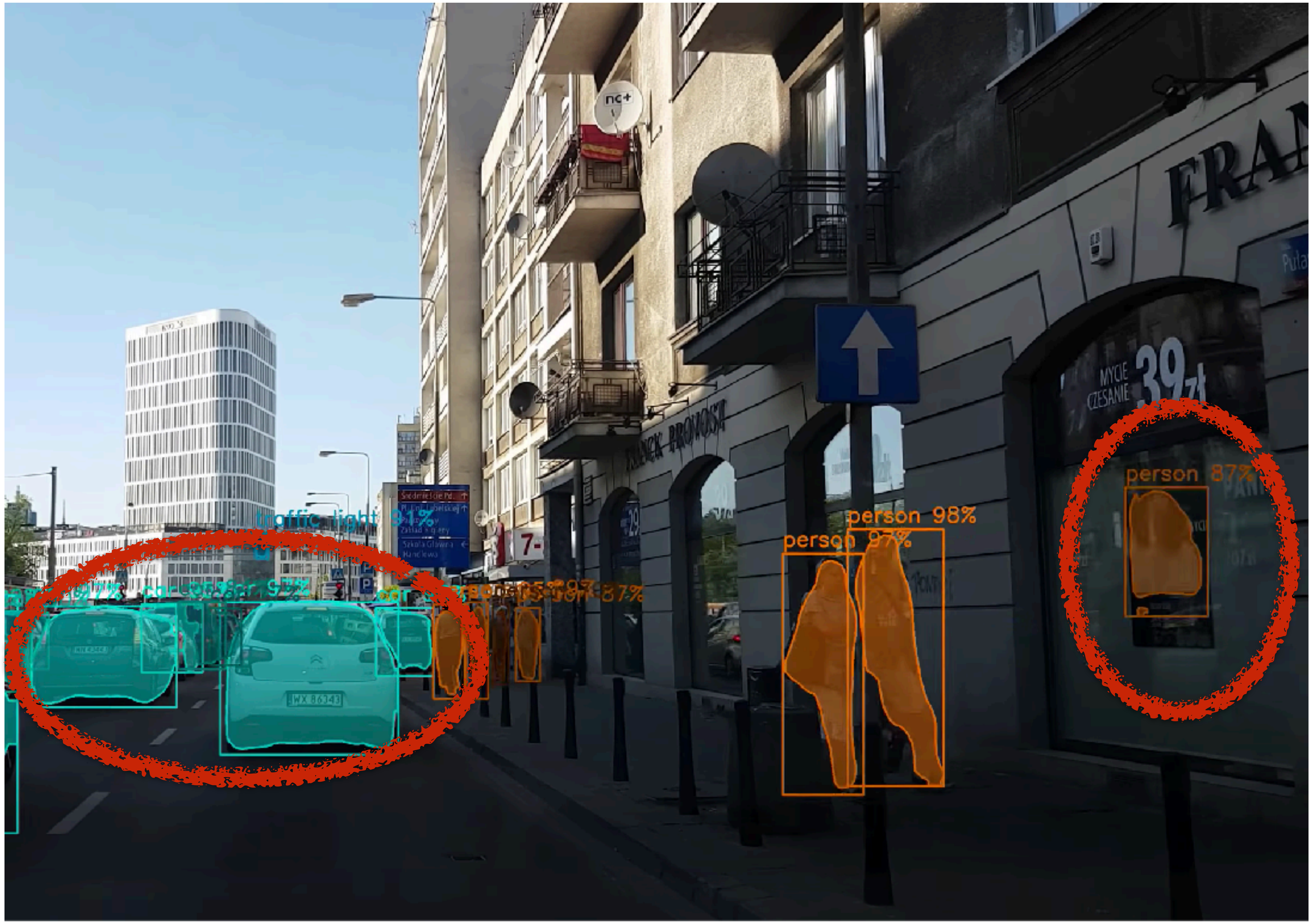

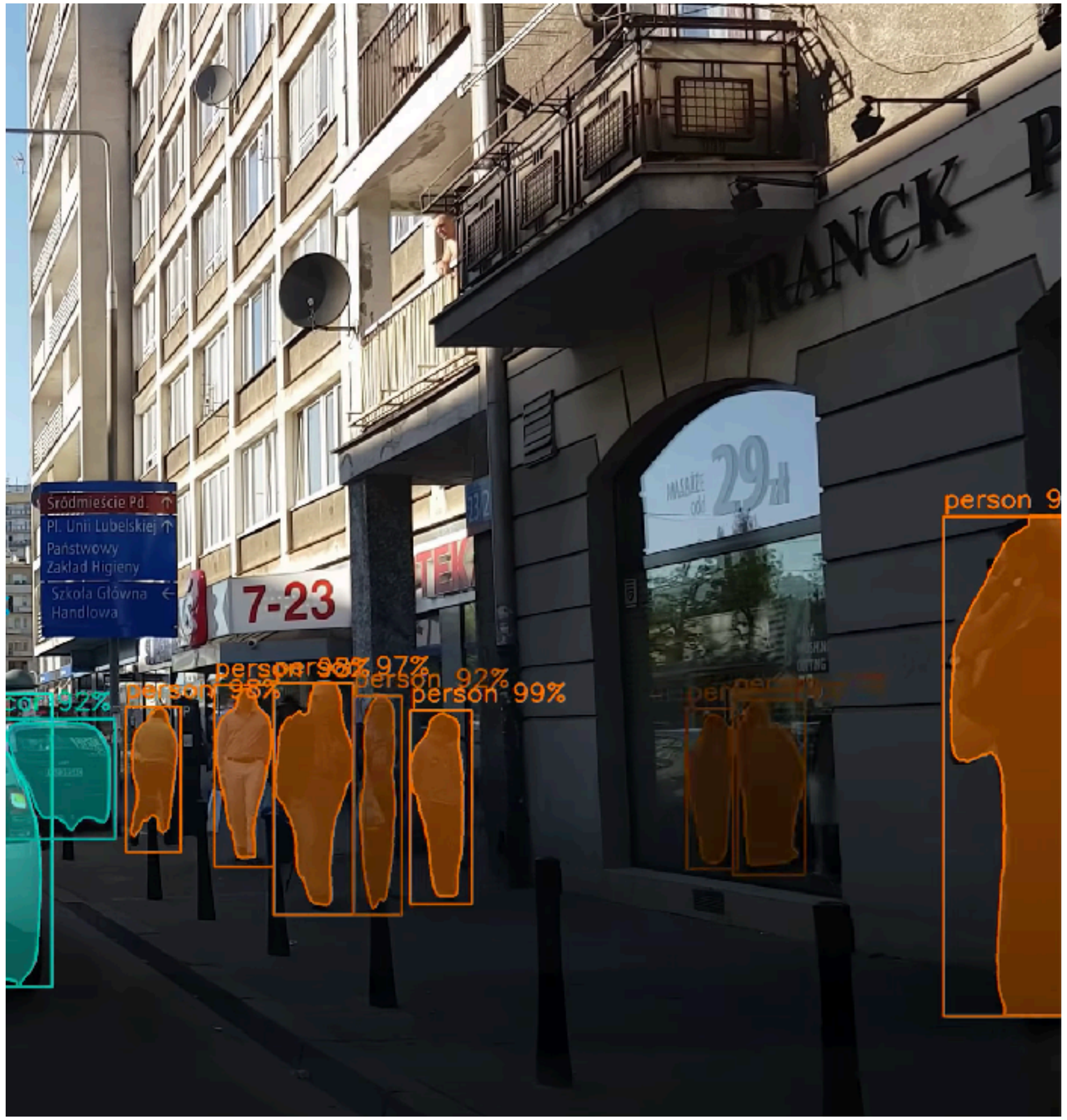

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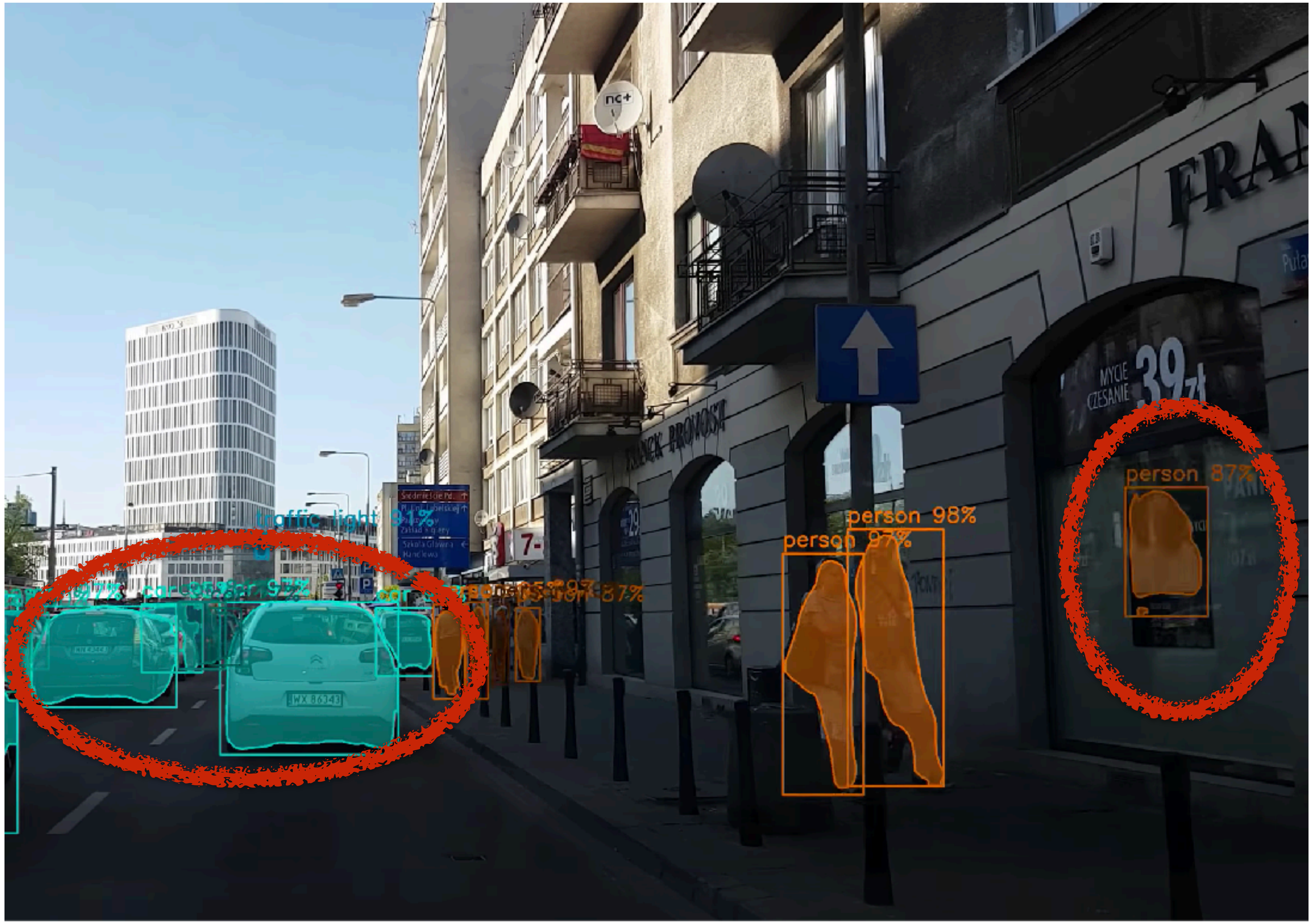

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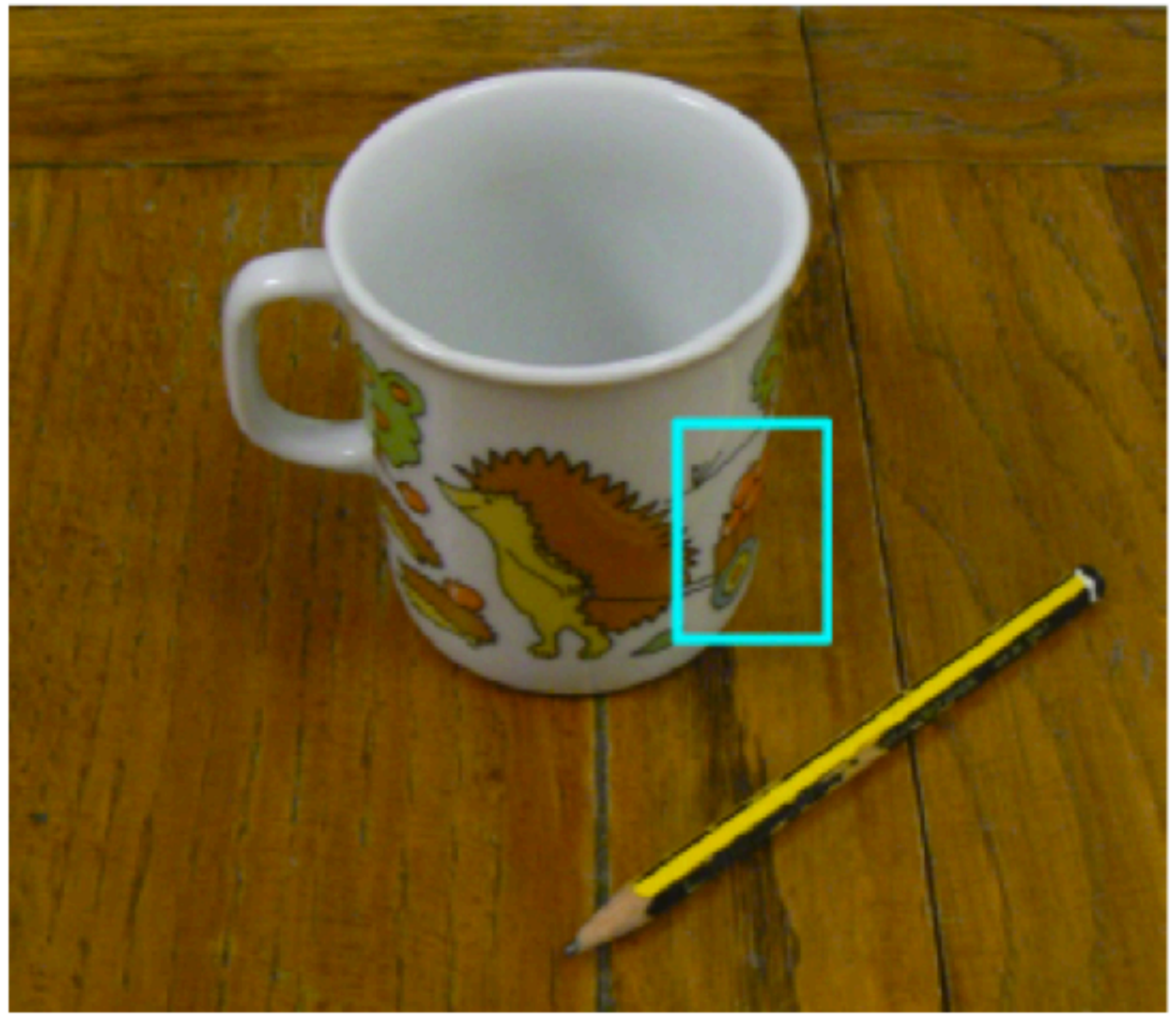
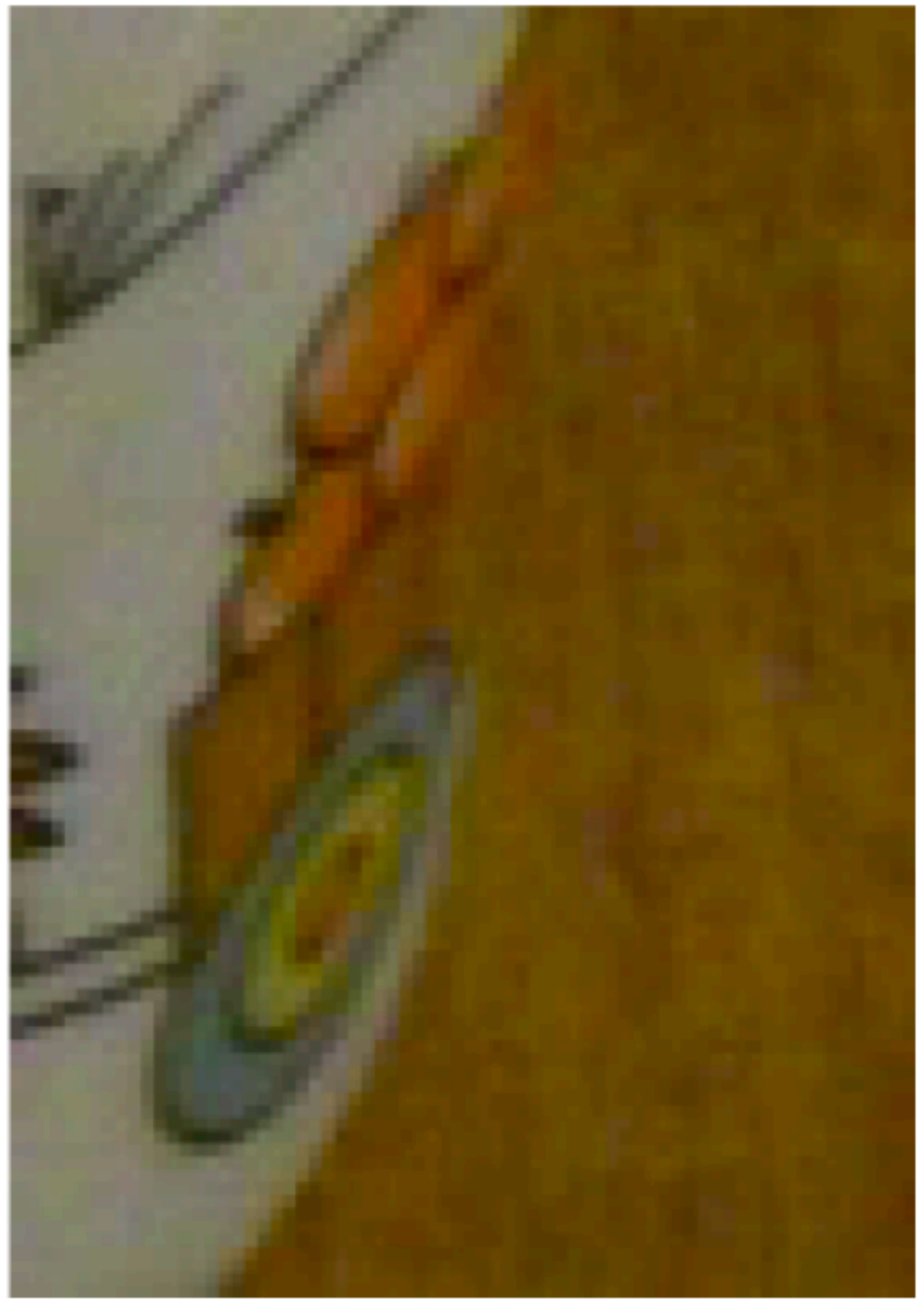
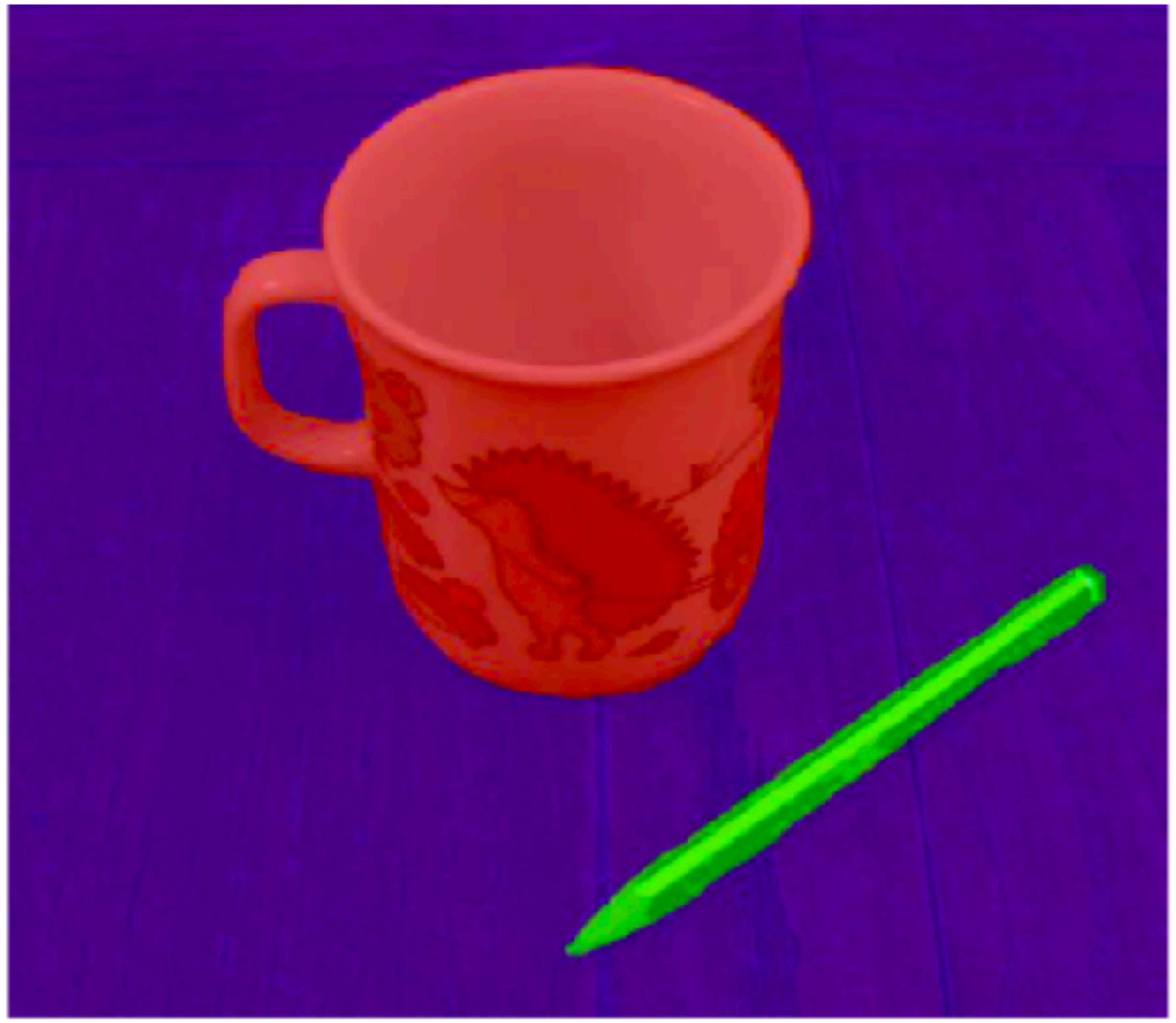




- Cannot use color to distinguish between border of cup and background
- Need some semantic understanding of what a “cup” is

slide credit: Václav Hlaváč

Image Segmentation As Grouping



Image, courtesy Ondřej Drbohlav

Goal: **group pixels** that belong together into **regions**

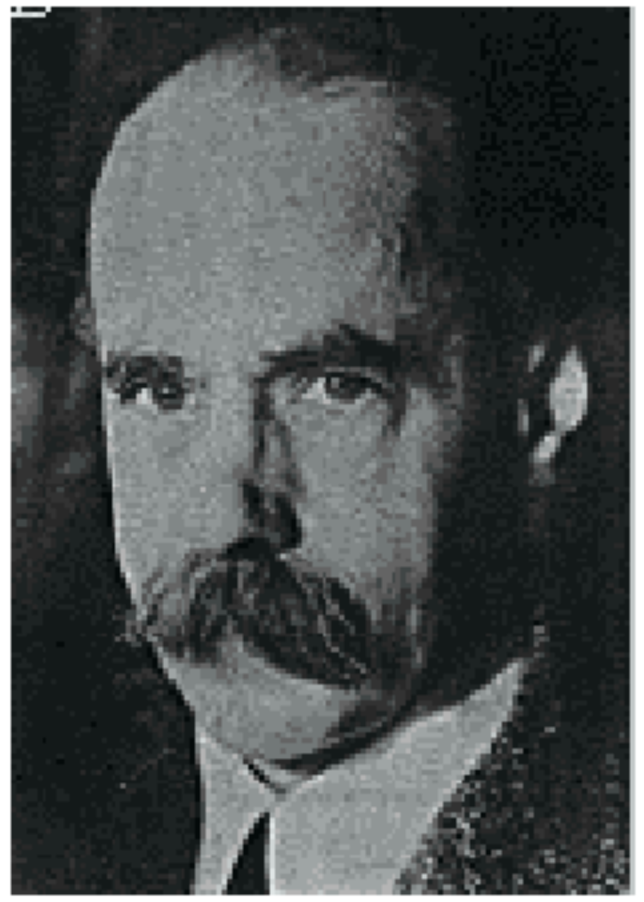
slide credit: Václav Hlaváč

Inspiration from Humans? - The Gestalt School

- **Grouping of elements is key to human visual perception**
- Founding publication by Max Wertheimer (born in Prague) in 1912
- Gestalt theory was meant to be generally applicable, but main tenets almost exclusively derived from observations of visual perception
- Psychologists showed that human visual systems seems predisposed to group elements

"I stand at the window and see a house, trees, sky. Theoretically I might say there were 327 brightnesses and nuances of colour. Do I have "327"? No. I have sky, house, and trees."

Max Wertheimer
(1880-1943)



Untersuchungen zur Lehre von der Gestalt, *Psychologische Forschung*, Vol. 4, pp. 301-350, 1923

slide credit: Václav Hlaváč, Bastian Leibe

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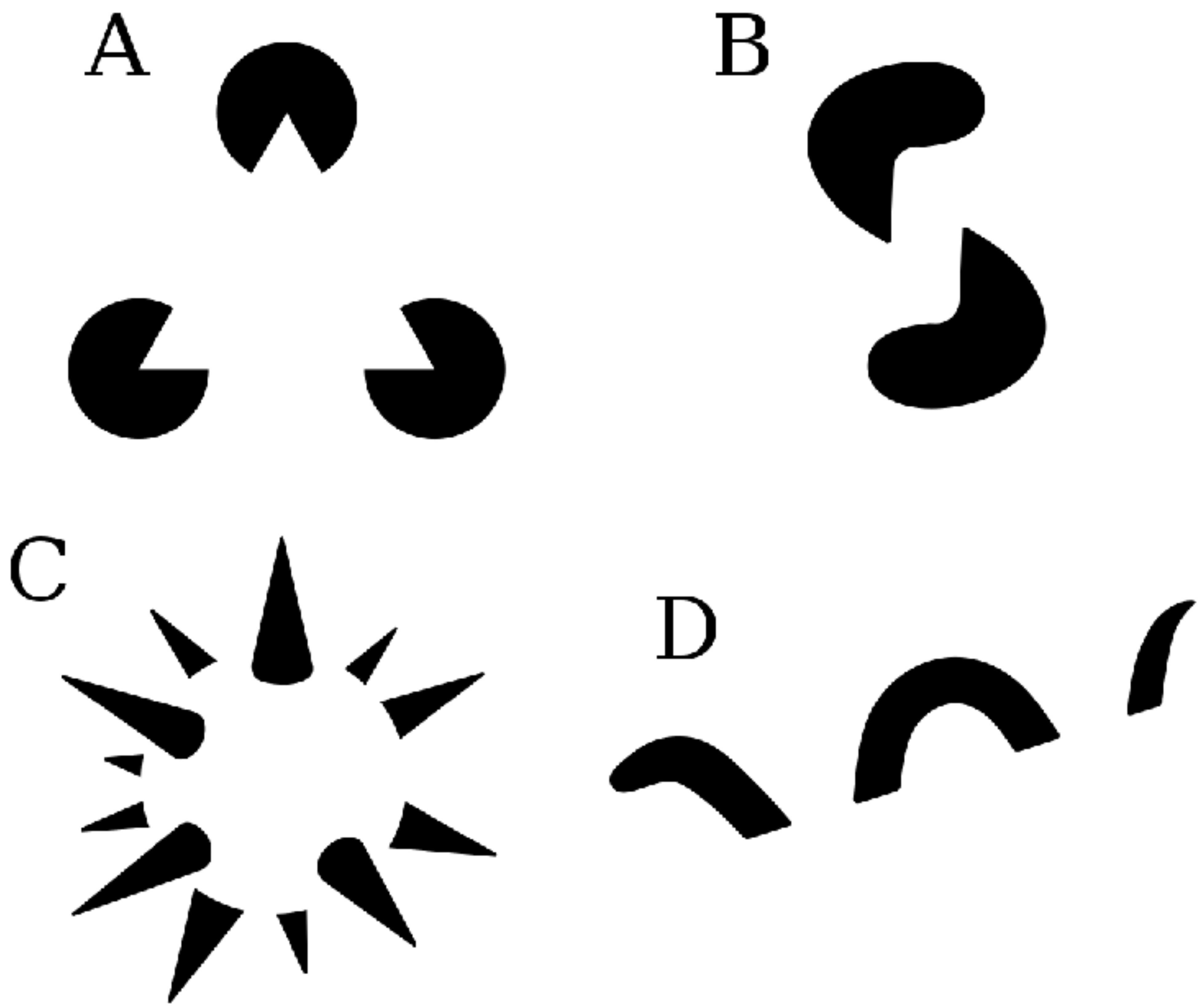
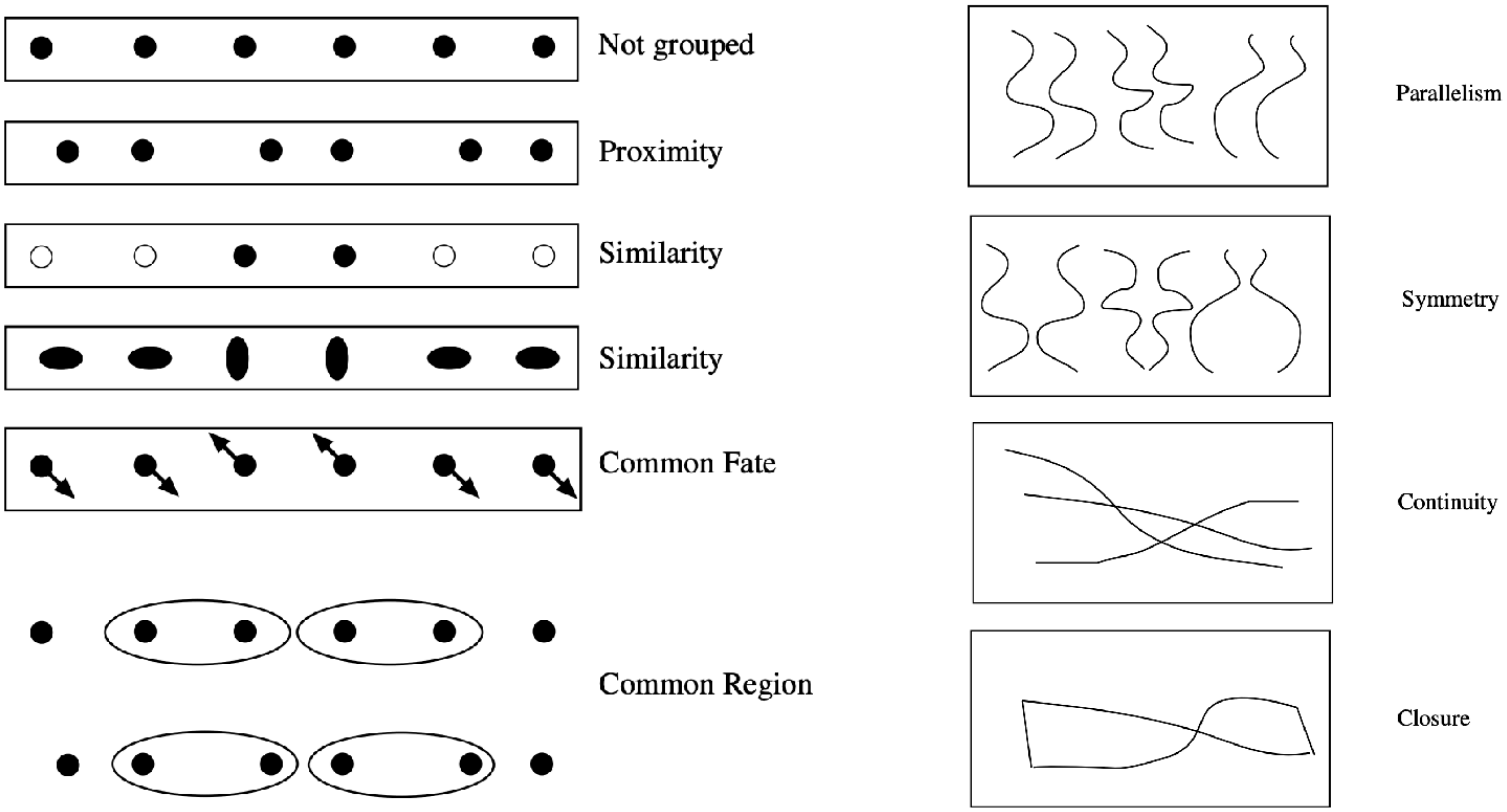


image source: Wikipedia

- **Gestalt**: configuration of elements such that whole is greater than sum of parts
- Properties / features derived from relationship between elements
- https://en.wikipedia.org/wiki/Gestalt_psychology

slide credit: Václav Hlaváč, Bastian Leibe

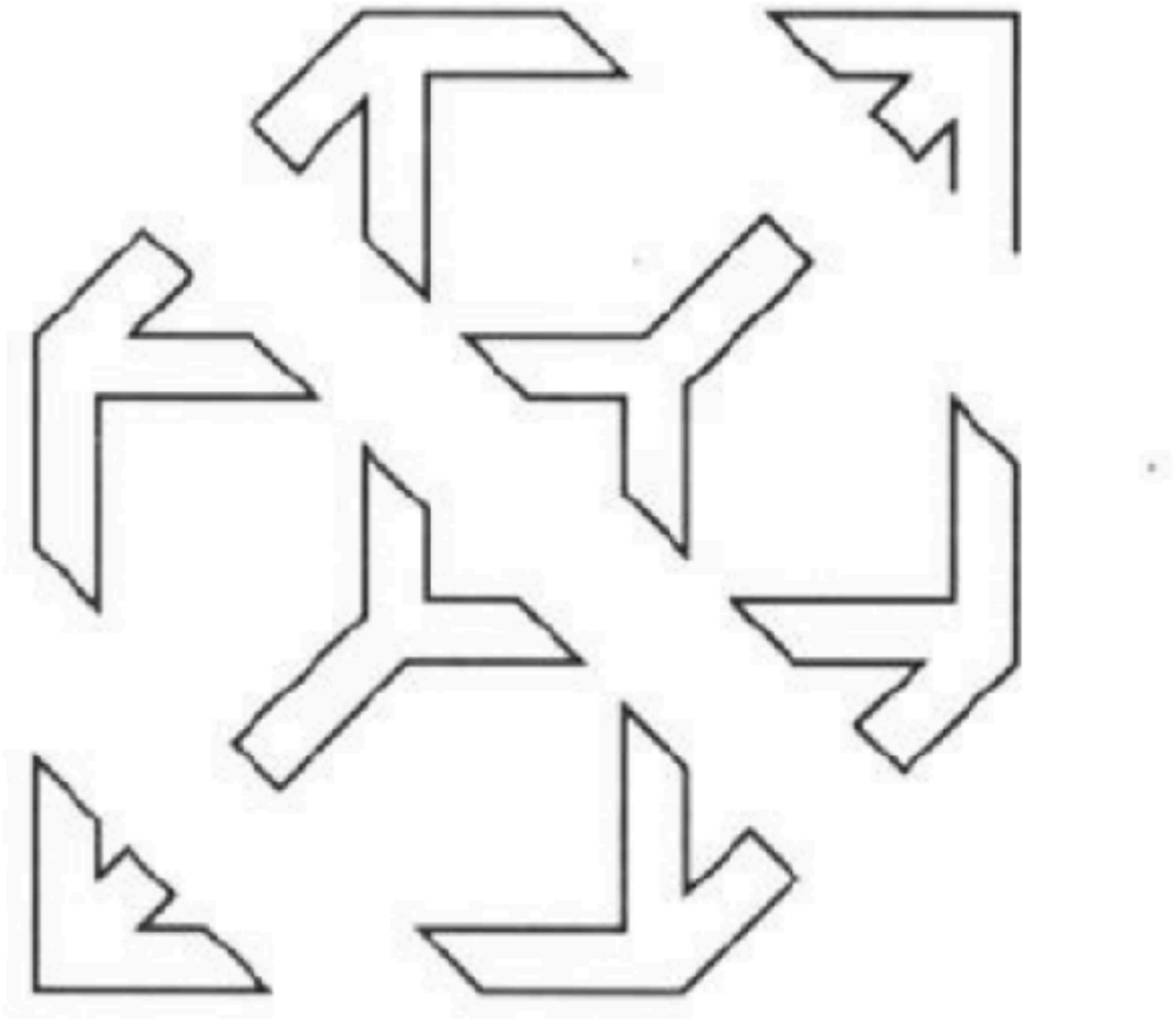
Gestalt Grouping Principles



slide credit: Václav Hlaváč, Bastian Leibe

image source: D. Forsyth, J. Ponce, Computer Vision - A Modern Approach, 2nd edition, Pearson, 2011

Continuity Through Occlusions



slide credit: Bastian Leibe

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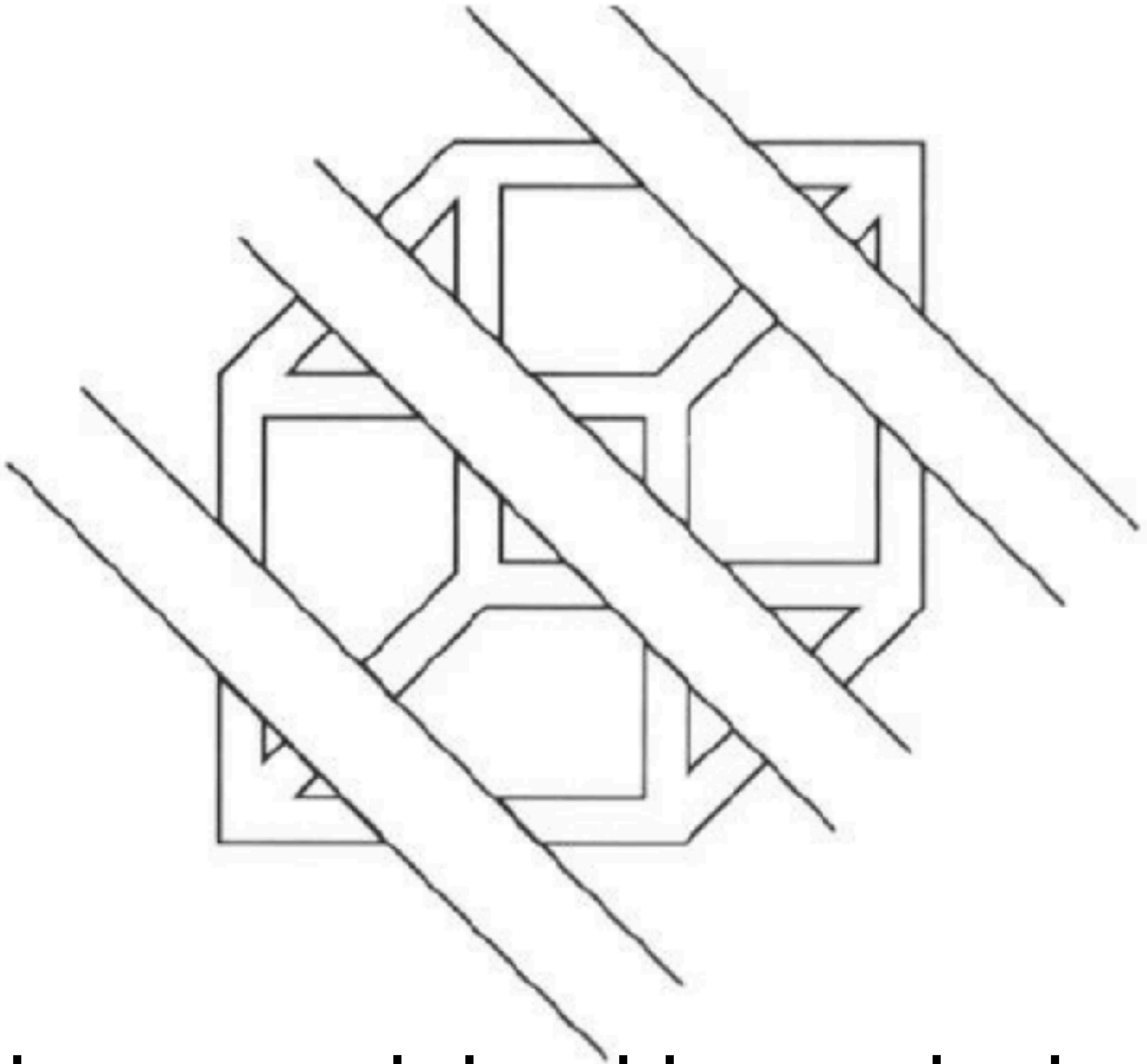
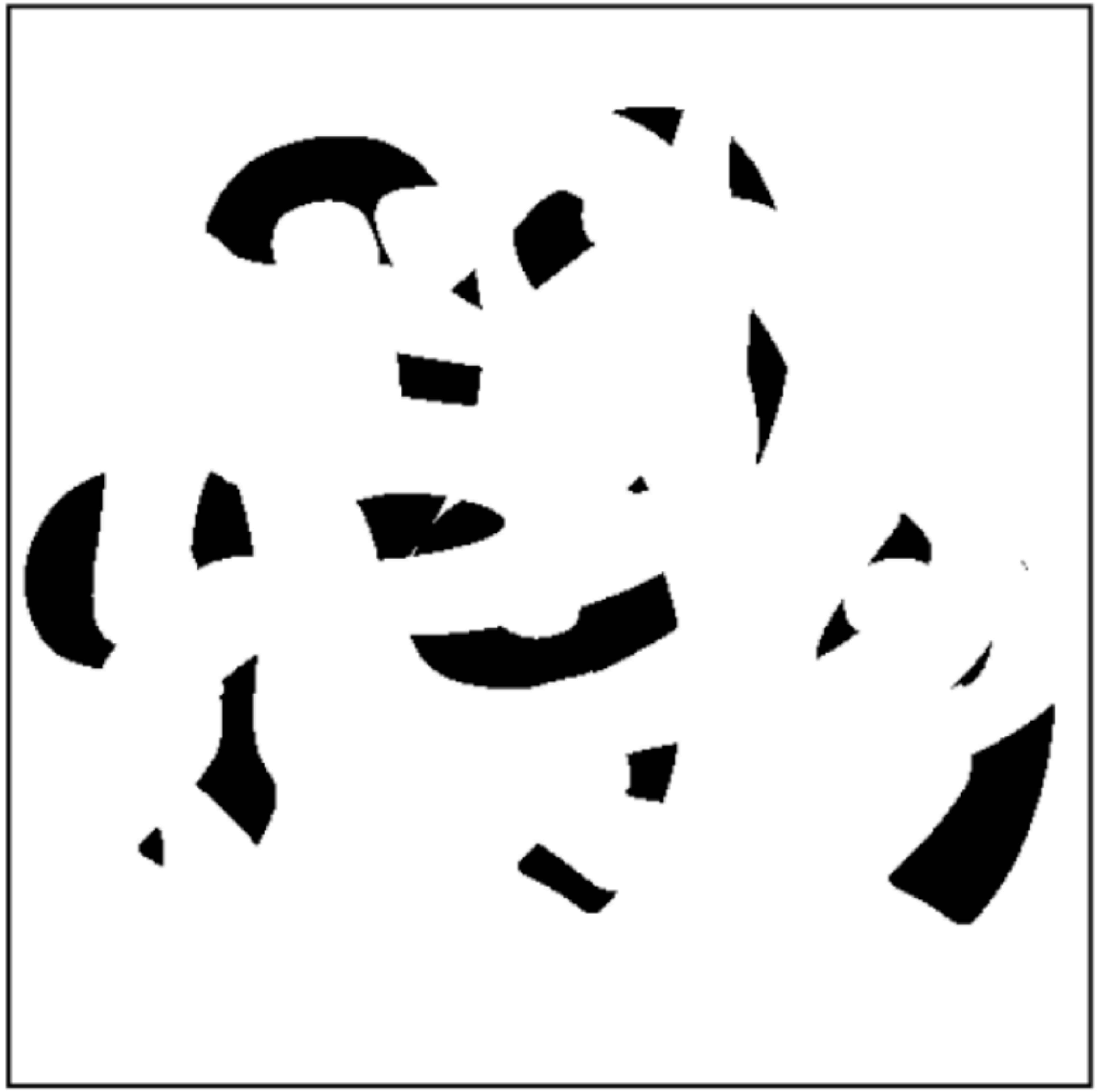


Shape explained by occlusions

slide credit: Bastian Leibe

Continuity Through Occlusions

What do you see?



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Grouping Can Be Very Hard

What do you see?

Picture by R. C. James

slide credit: Václav Hlaváč, Bastian Leibe

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How to teach Gestalt principles to a machine?

slide credit: Václav Hlaváč, Bastian Leibe

Lecture Overview

- A simple approach to segmentation: **(intensity) thresholding**

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Lecture Overview

simple & heuristic

complex & principled

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Other Segmentation Approaches Not Covered

- **Template matching**: detect regions in image by comparing with templates, fitting structures in the image
  - Object detection based on templates
  - Parametric model detection, e.g., straight lines, circles, ellipses, …

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  - Object detection based on templates
  - Parametric model detection, e.g., straight lines, circles, ellipses, …
- **Based on unusual phenomena**: segmentation by detecting unusual structures
  - Camouflage detection based on unusual texture
  - Image compression: large regions as unusual occurrences that can be heavily compressed (e.g., regions of same color)

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A Few Words of Advice

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- There is **no general purpose** segmentation algorithm for all cases
- Algorithm to use depends on circumstances:
  - Lots of labelled training data → supervised learning with CNNs
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- Use **prior knowledge** whenever available:
  - Knowledge about shape or color of an object
  - Priors on position of object or region in image (e.g., images centered on object)
  - Relation between objects or regions (e.g., car always on top of road)

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Not Covered: Feature Design

- We directly observe **primary features**: pixel intensities, colors, depth (range cameras, e.g., LiDAR, Kinect), temperature (thermal cameras)

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- How do we compare pixels / structures / regions? Extract **features** from direct observations:
  - Primary features (typically not very robust, e.g., to illumination changes)
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- **Modern choice**: learn features from data → deep learning / machine learning

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Image Segmentation

- Goal: compute **complete segmentation** of image
  - Subdivide the image $\mathcal{I}$ into S disjoint regions R_1, R_2, …, R_s, i.e.,

$$\mathcal{I} = \bigcup_{i=1}^{S} R_i, \quad R_i \cap R_j = \emptyset, i \neq j$$

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- Simplest case: **binary segmentation** into **foreground** (objects) and **background**
  - Surprisingly often a valid assumption as we often do not care about background

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Image Segmentation via Thresholding

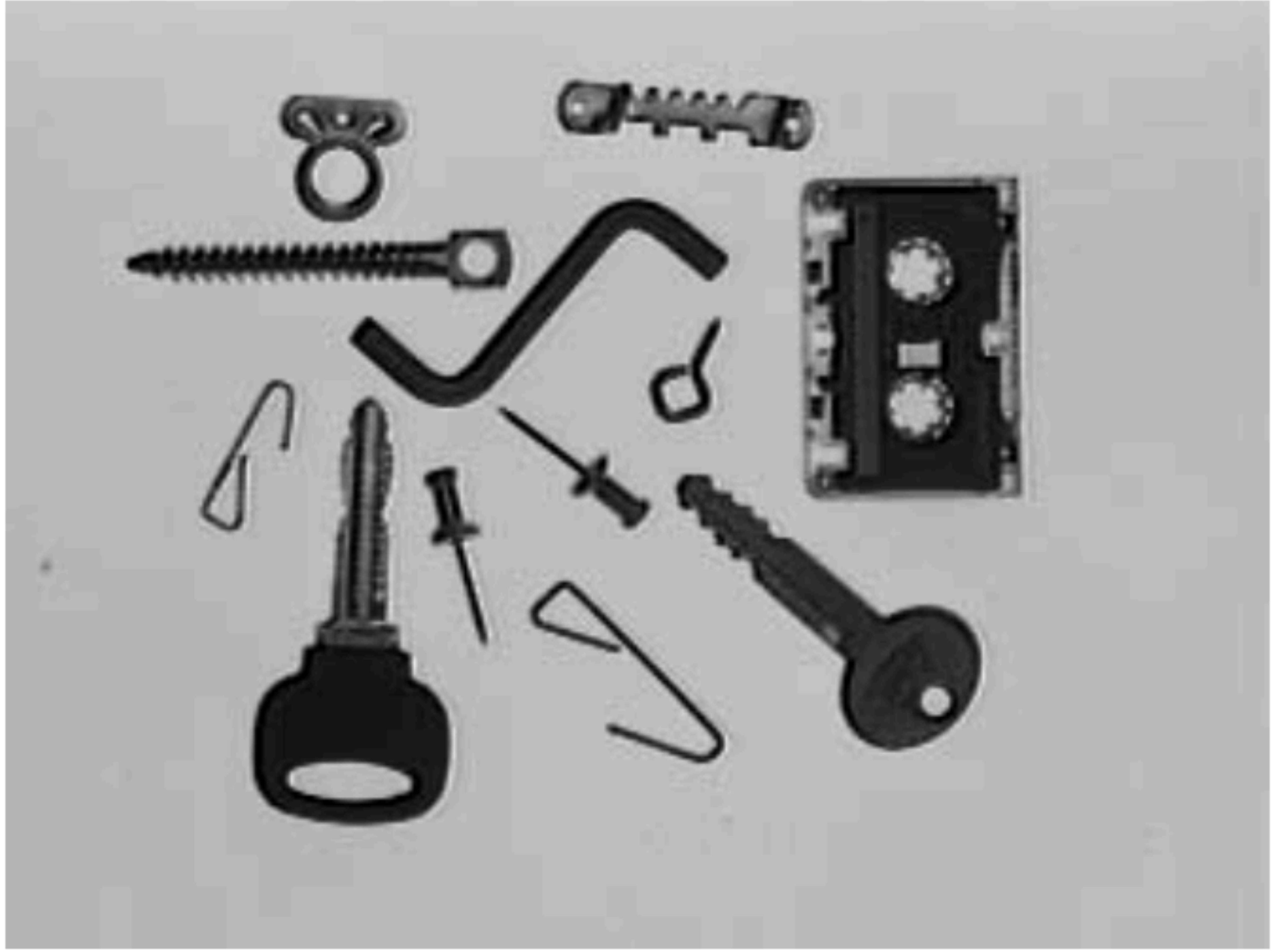

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Image Segmentation via Thresholding

simple background

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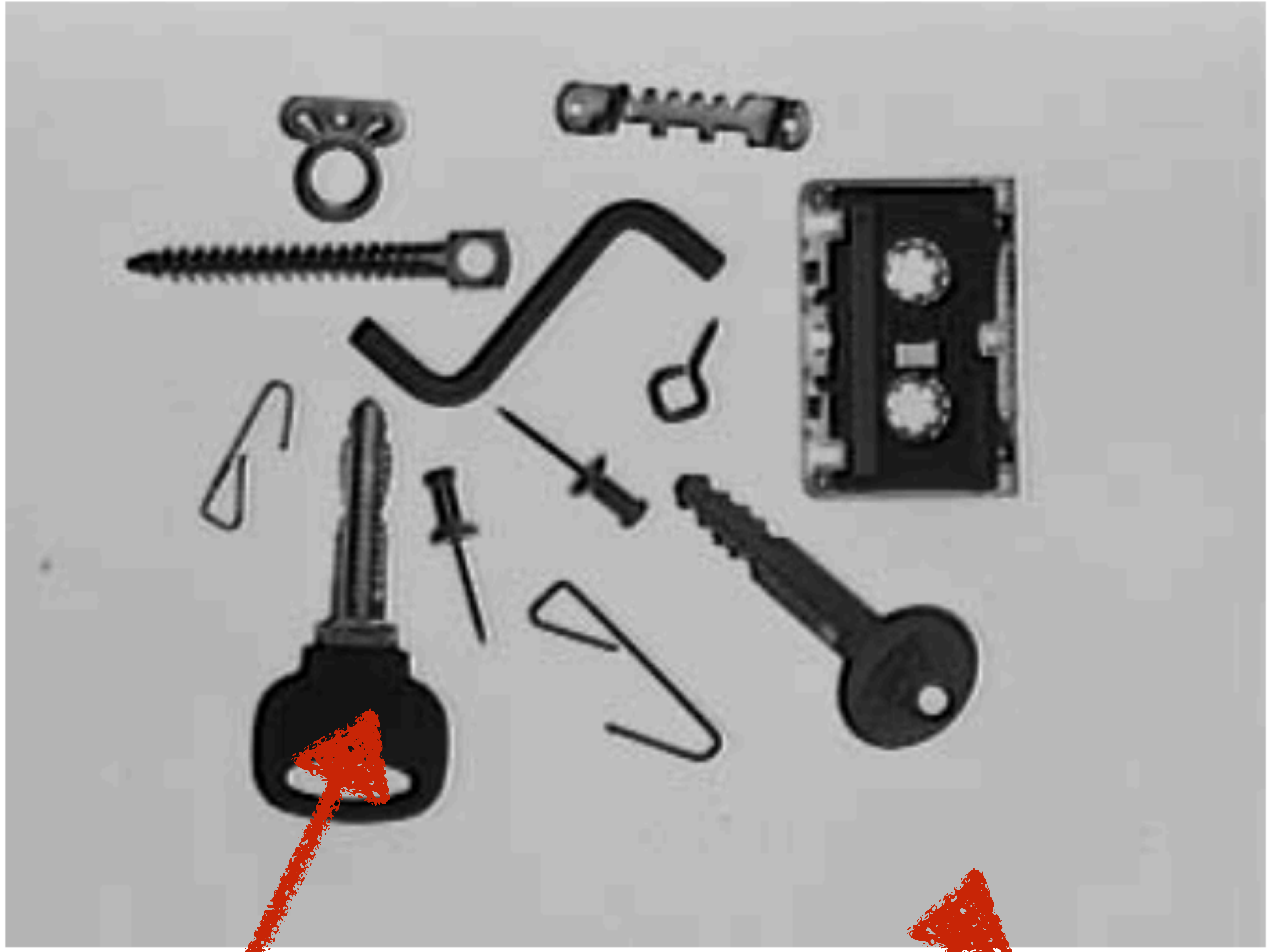

distinctly colored objects

simple background

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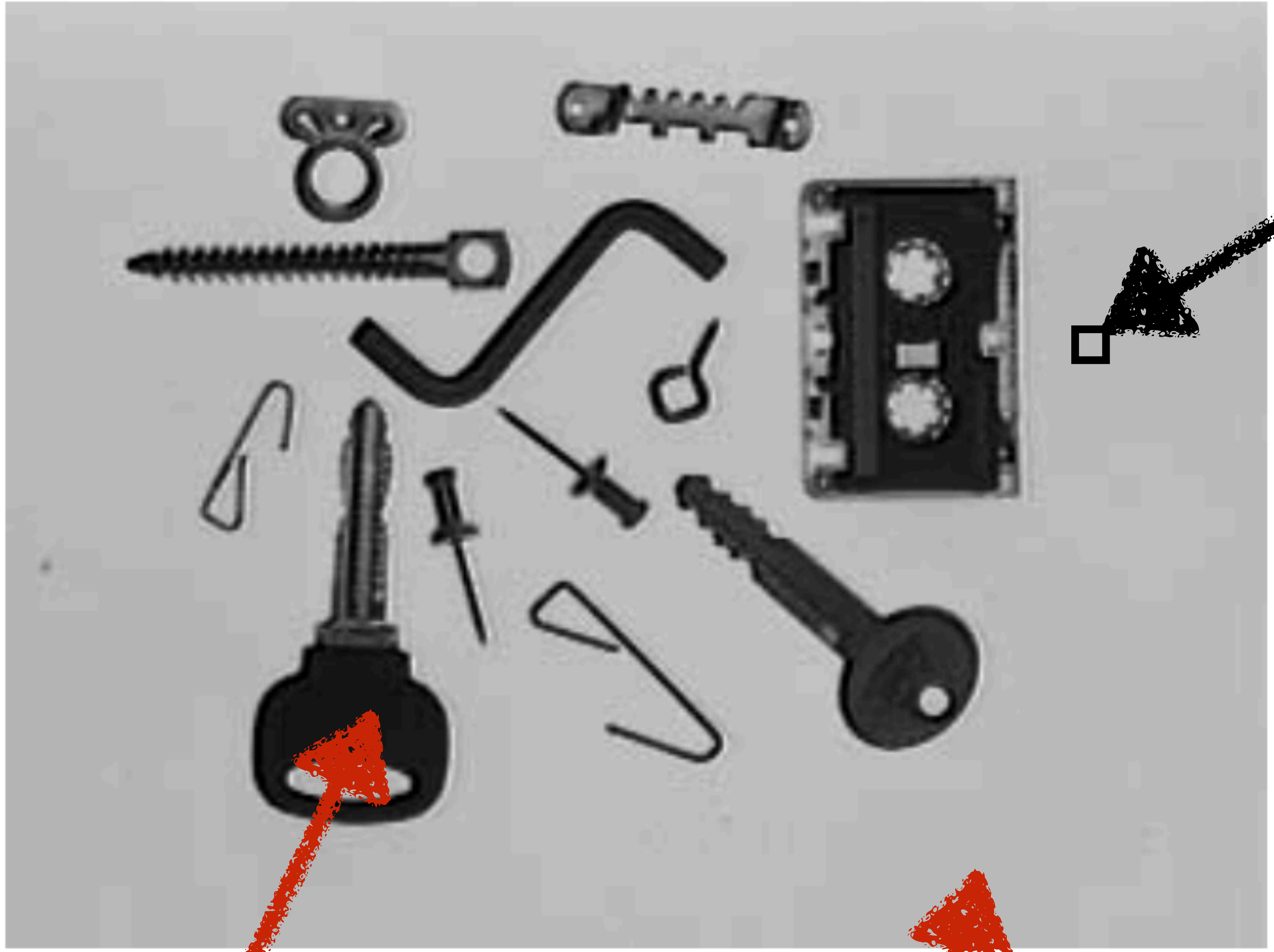

pixel (i, j) with intensity $f(i, j)$

distinctly colored objects

simple background

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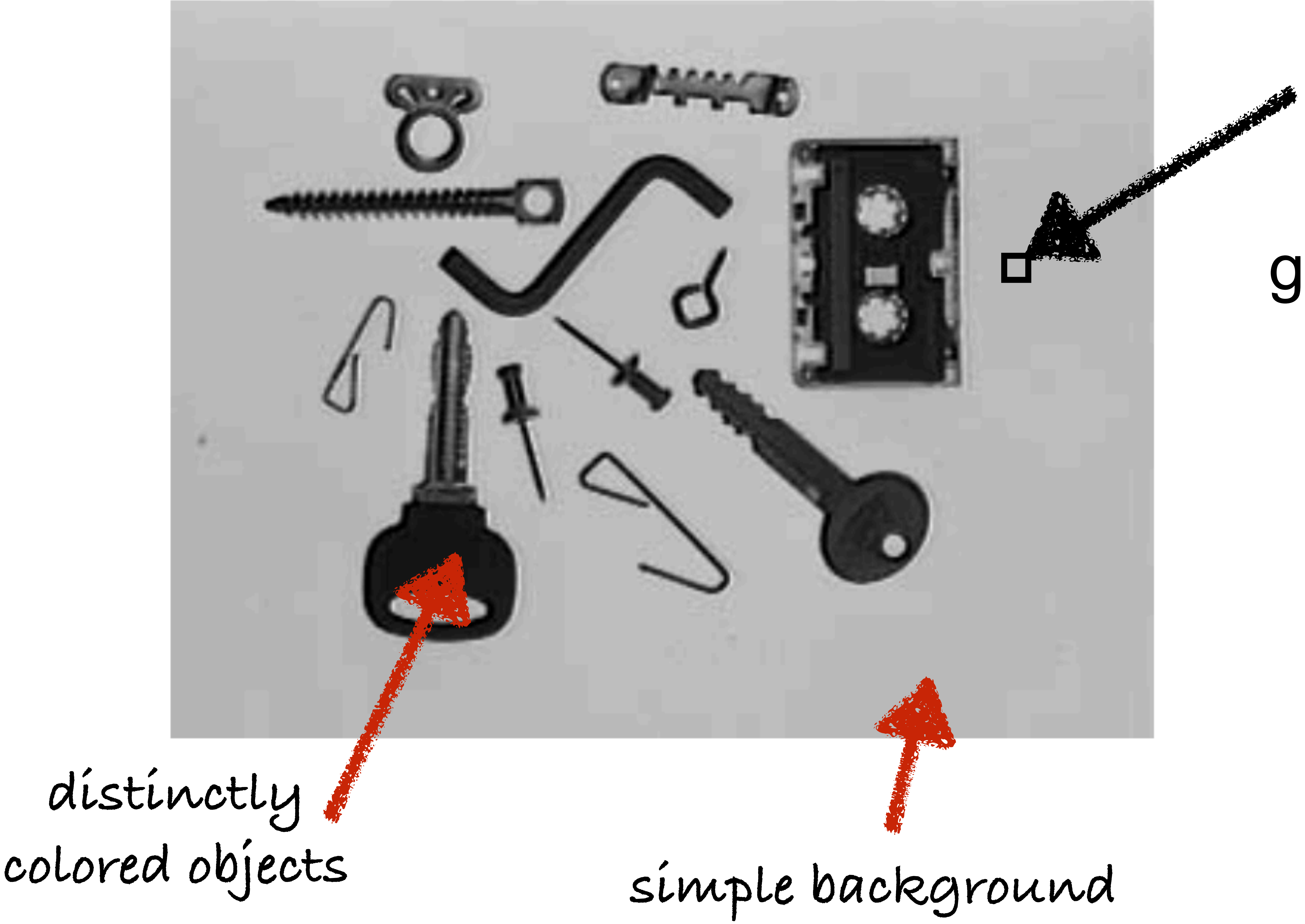

pixel (i,j) with intensity $f(i,j)$

generate binary image b with

$$g(i,j) = \begin{cases} 1 & \text{if} \quad f(i,j) \geq T \\ 0 & \text{if} \quad f(i,j) < T \end{cases}$$

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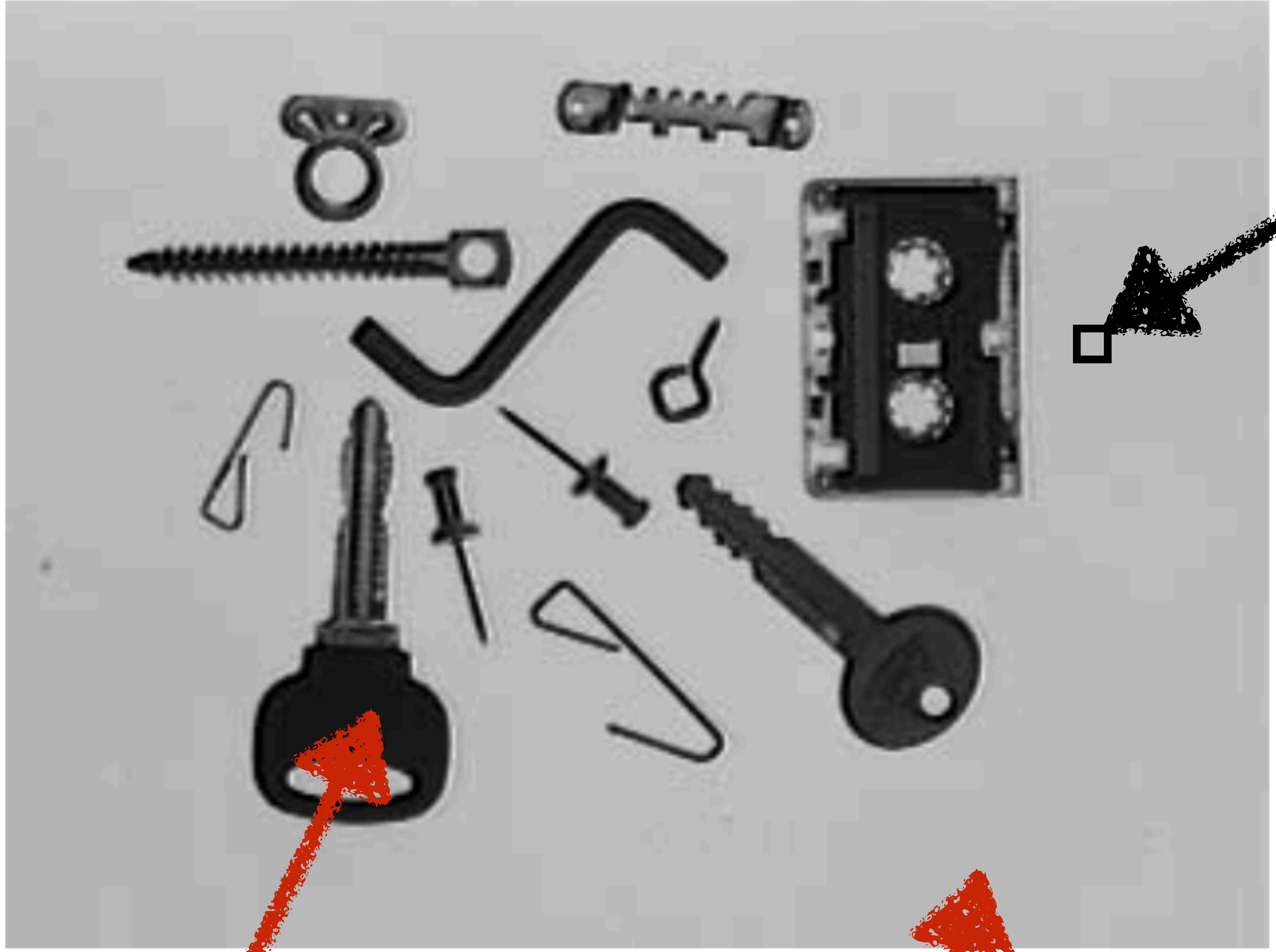

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threshold

distinctly colored objects

simple background

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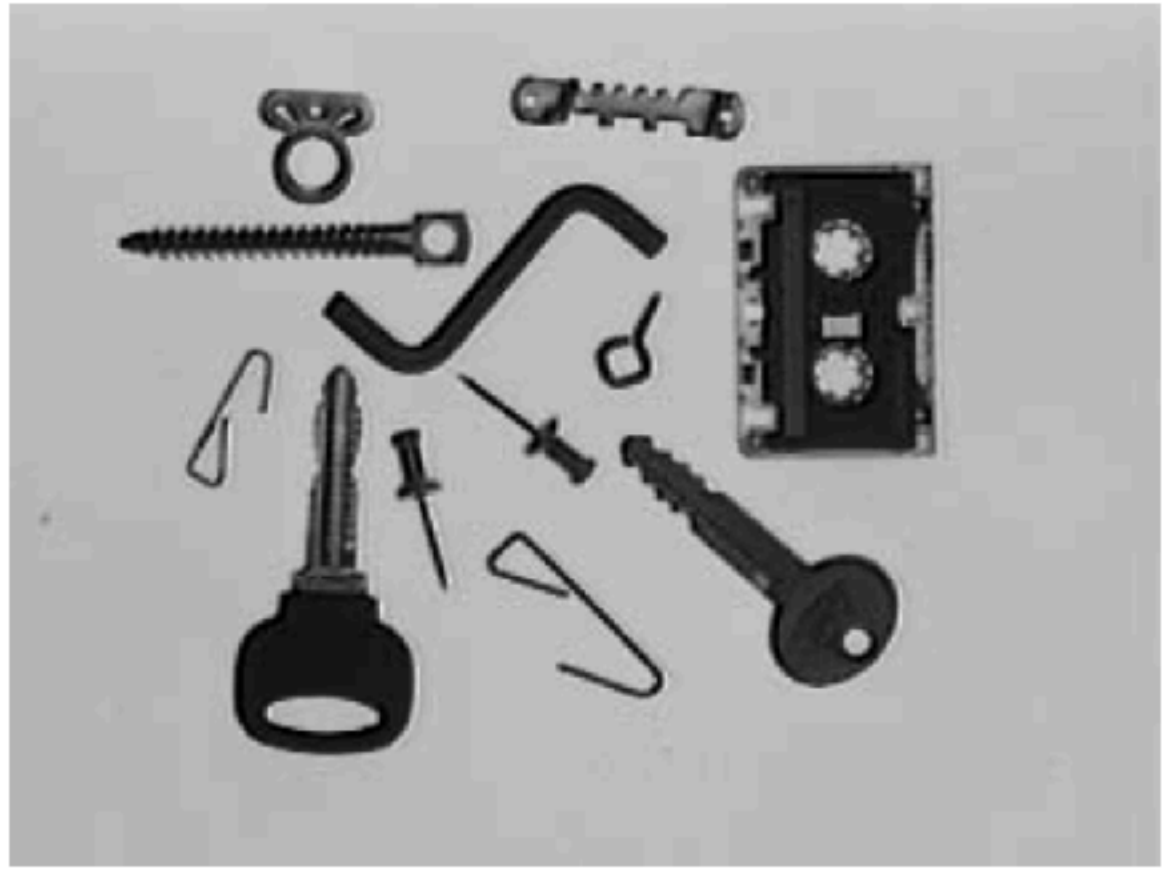
Example



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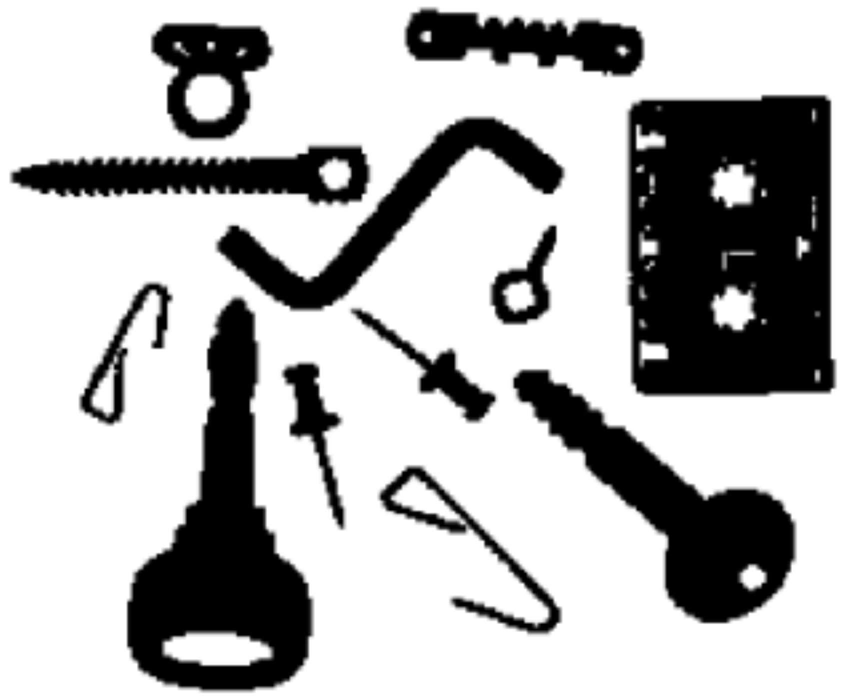
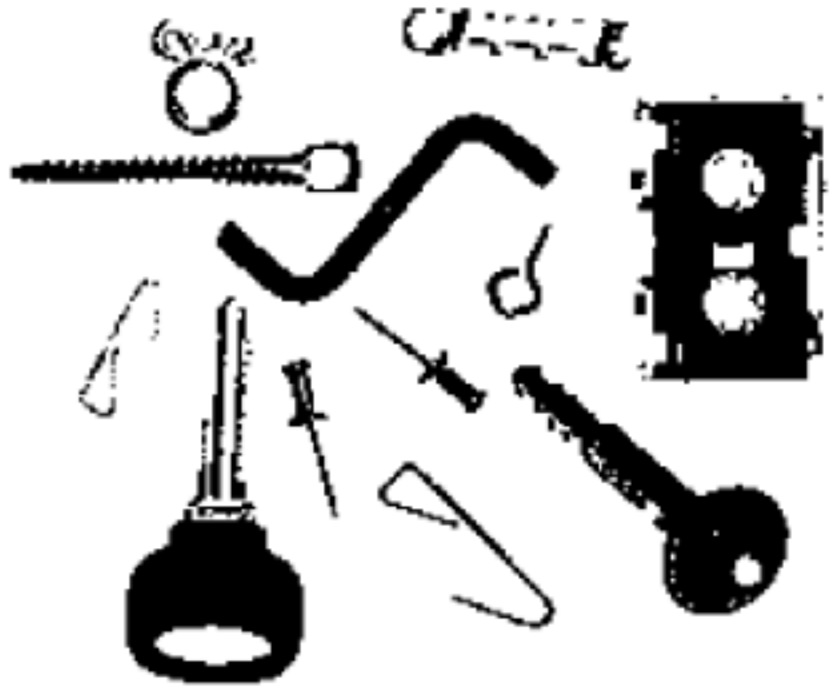
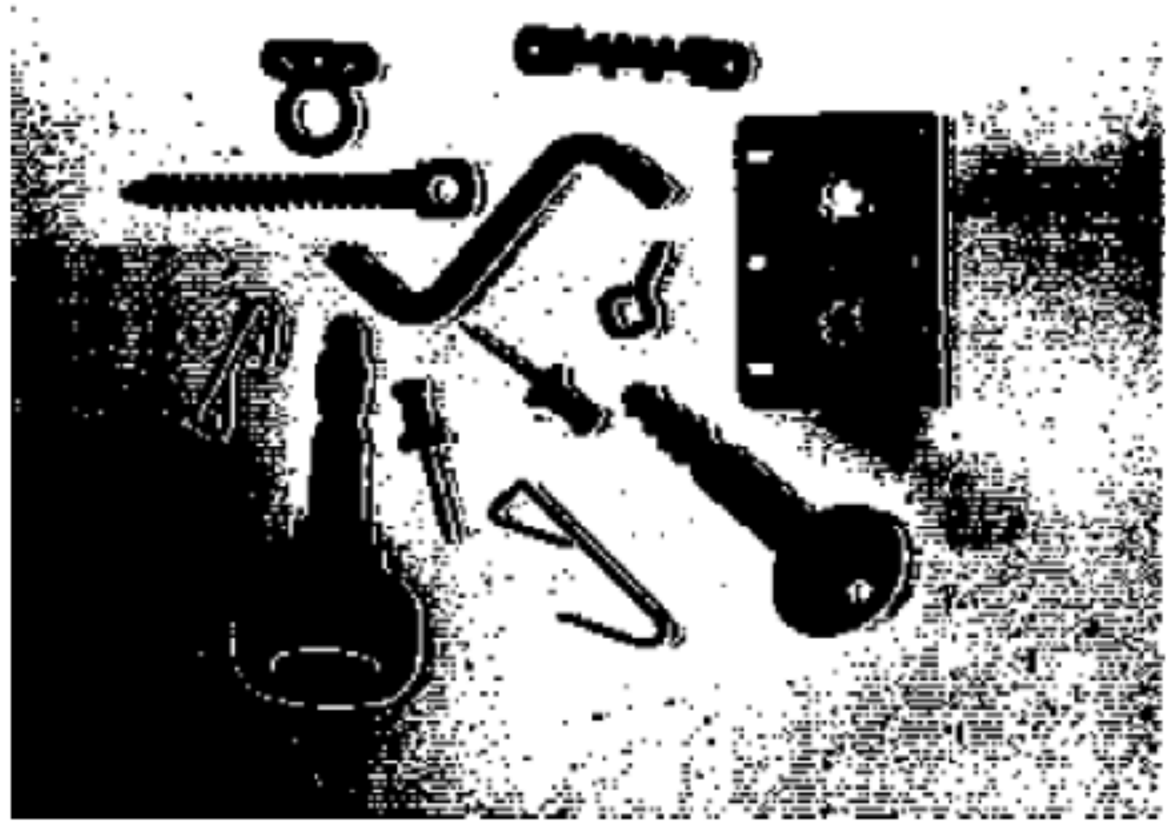


Threshold segmentation.



Threshold too low.



Threshold too high.

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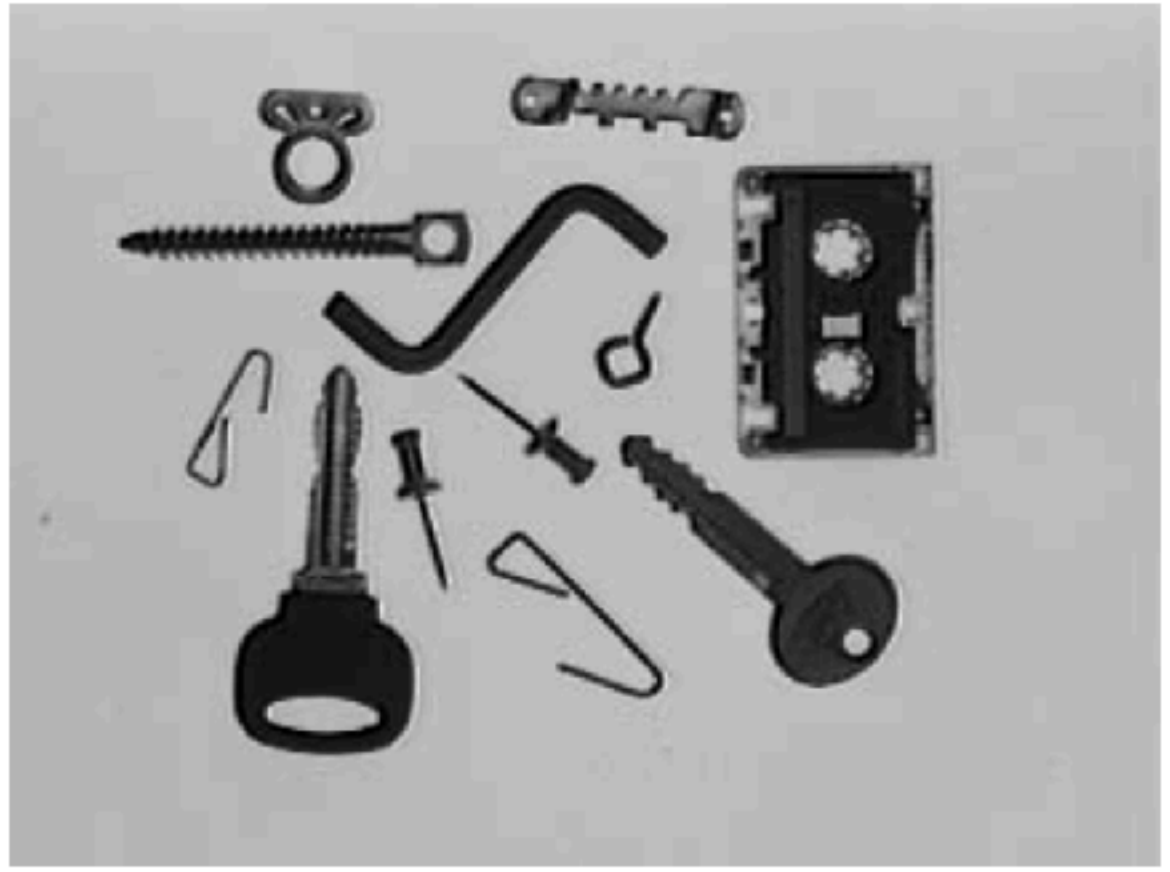

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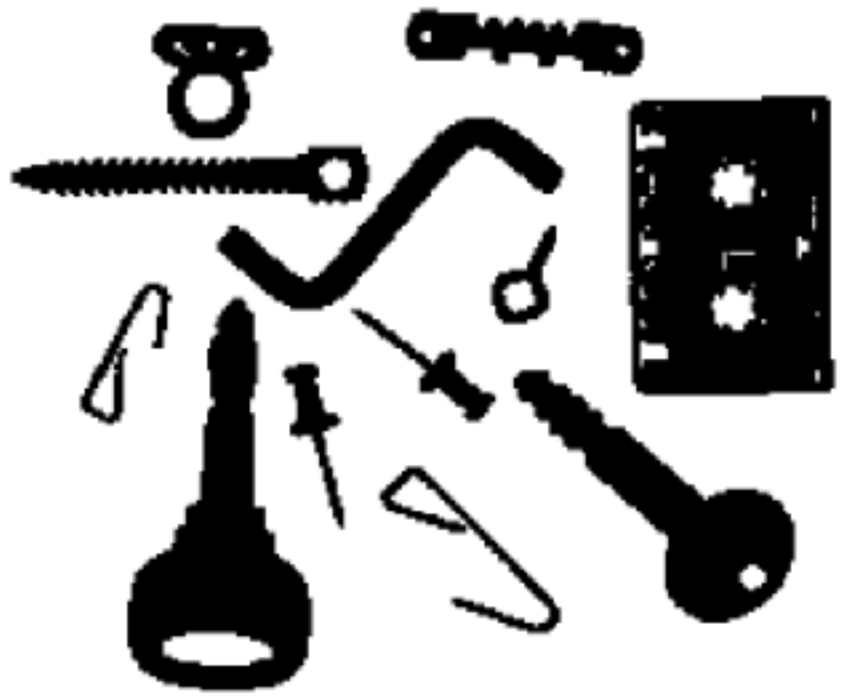

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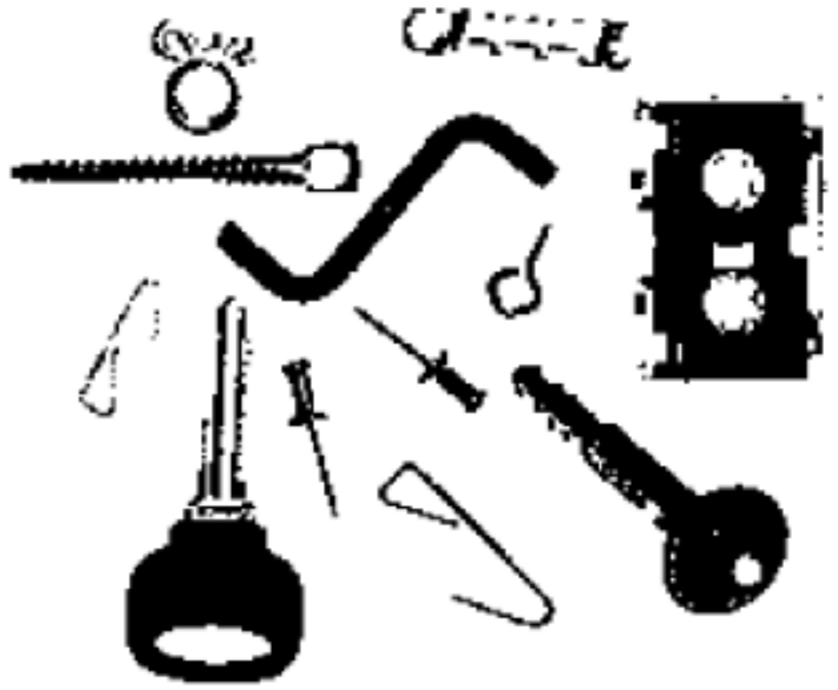

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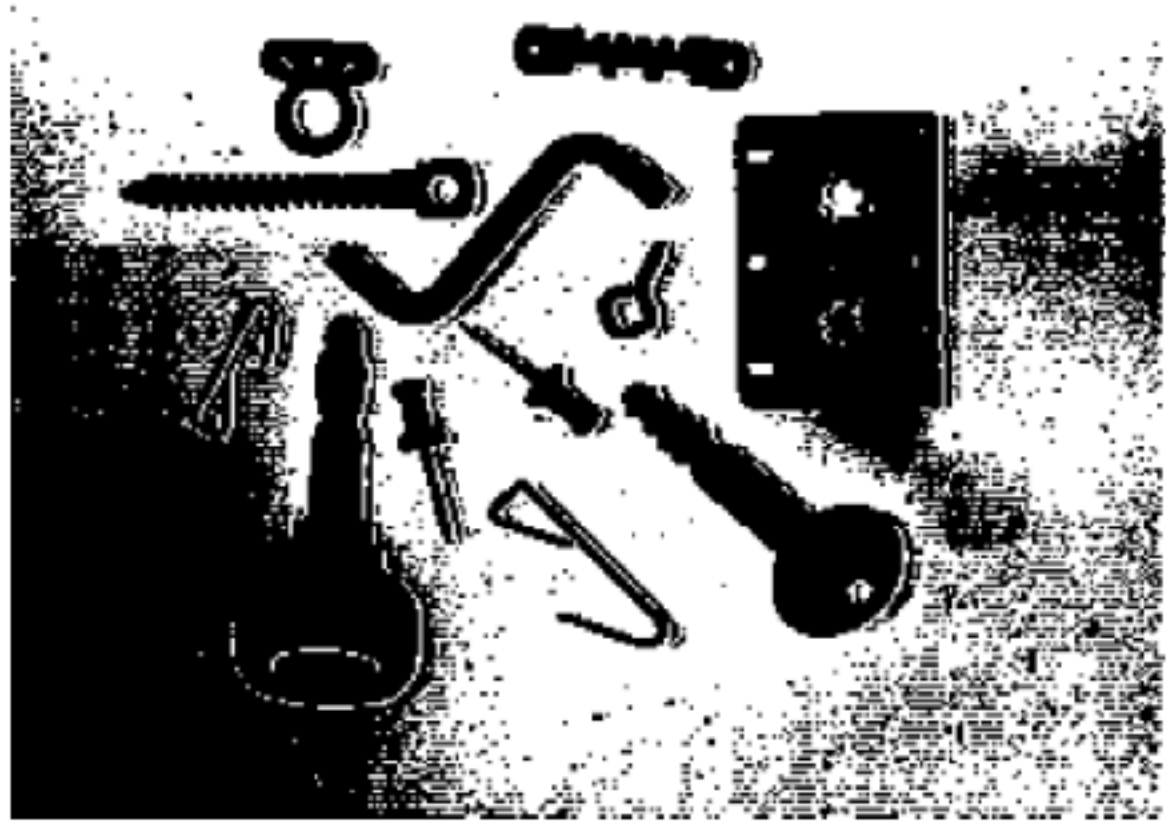

Threshold too high.

How to choose the threshold?

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Thresholding Choices

- **Band thresholding**: consider range D of intensities

$$g(i,j) = \begin{cases} 1 & \text{if} \quad f(i,j) \in D \\ 0 & \text{otherwise} \end{cases}$$

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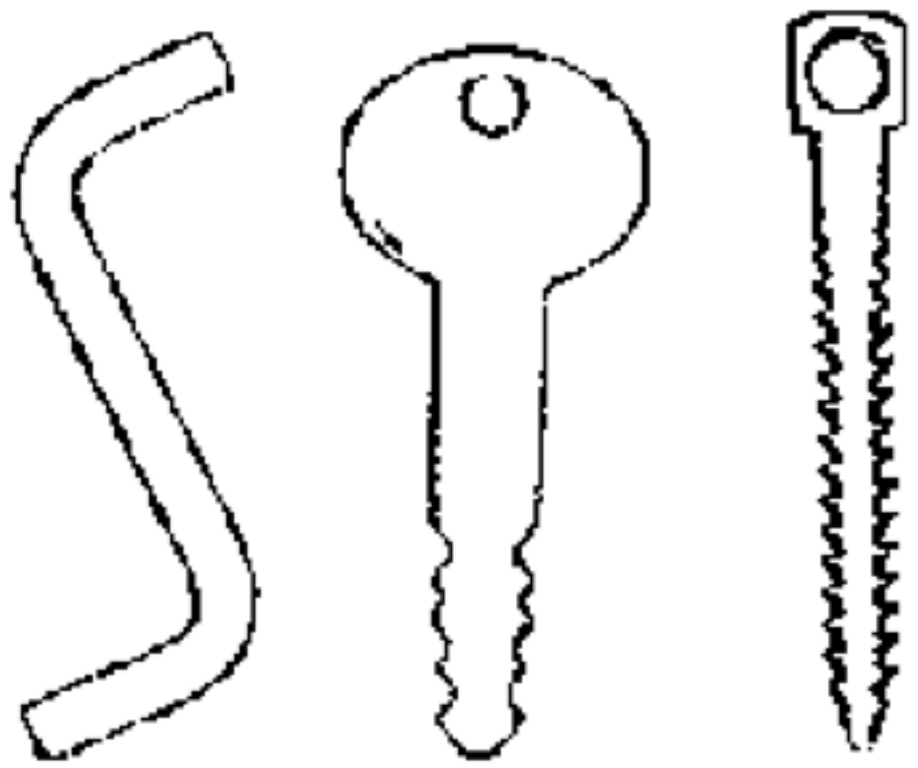

Border regions detected.

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- **Locally adaptive thresholding**: divide images into regions (e.g., regular grid) and find a threshold for each region

- **Multiple thresholds**: use multiple thresholds for $S > 2$ classes

$$g(i,j) = \begin{cases} 2 & \text{if} \quad f(i,j) \geq T_2 \\ 1 & \text{if} \quad T_1 \leq f(i,j) < T_2 \\ 0 & \text{if} \quad f(i,j) < T_1 \end{cases}$$

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- **Semi-thresholding**: only segment out the background, let human / other algorithm deal with foreground

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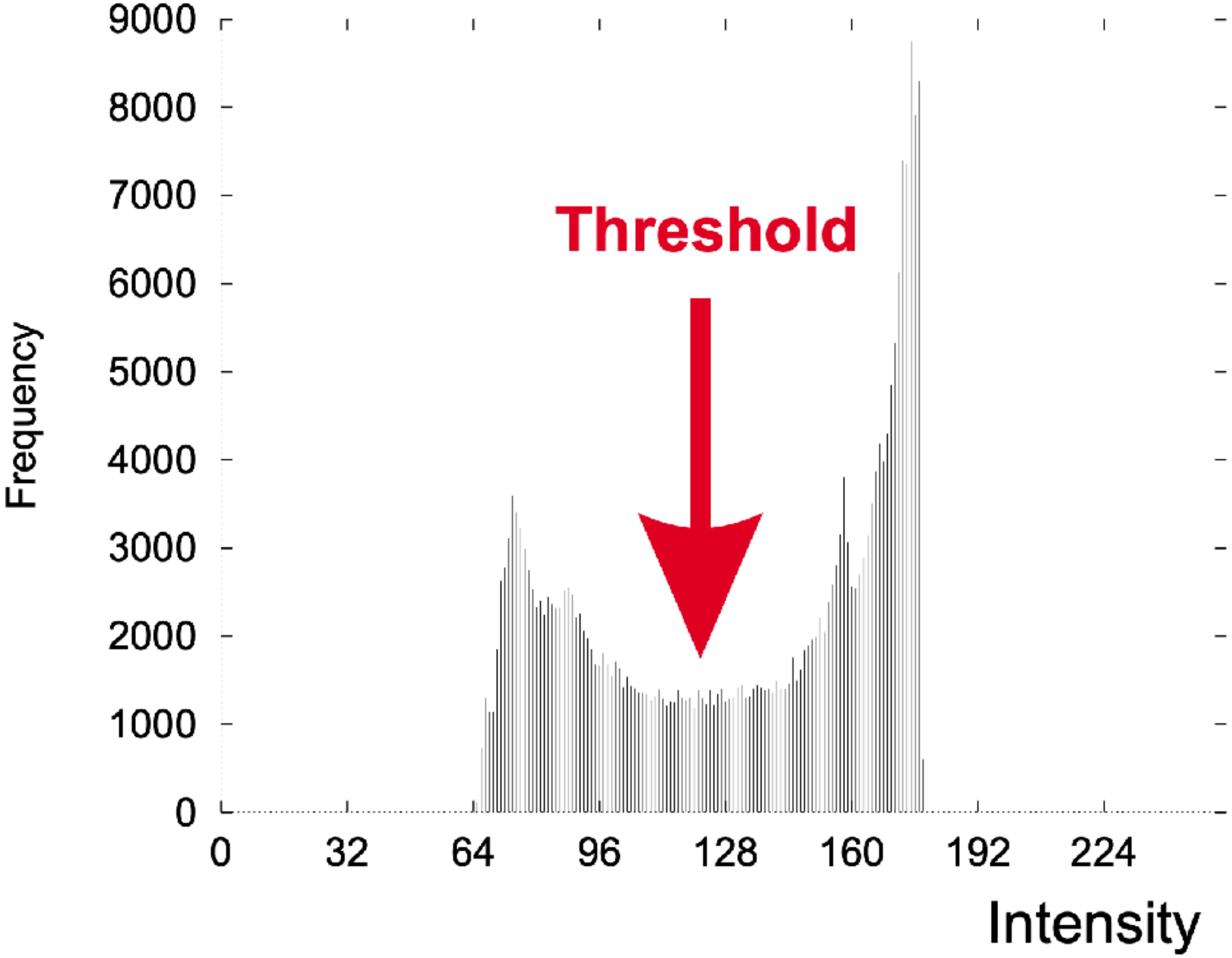
- ***p*-tile thresholding**: if object covers 1/*p* of image, find the corresponding 1/*p* of histogram (e.g., when we know size of printed characters)
- **Automatic thresholding based on histograms**: compute histogram of intensities, objects and background should correspond to distinct modes, find threshold(s) separating the modes

slide credit: Václav Hlaváč

Automatic Thresholding Based On Histograms



slide credit: Václav Hlaváč

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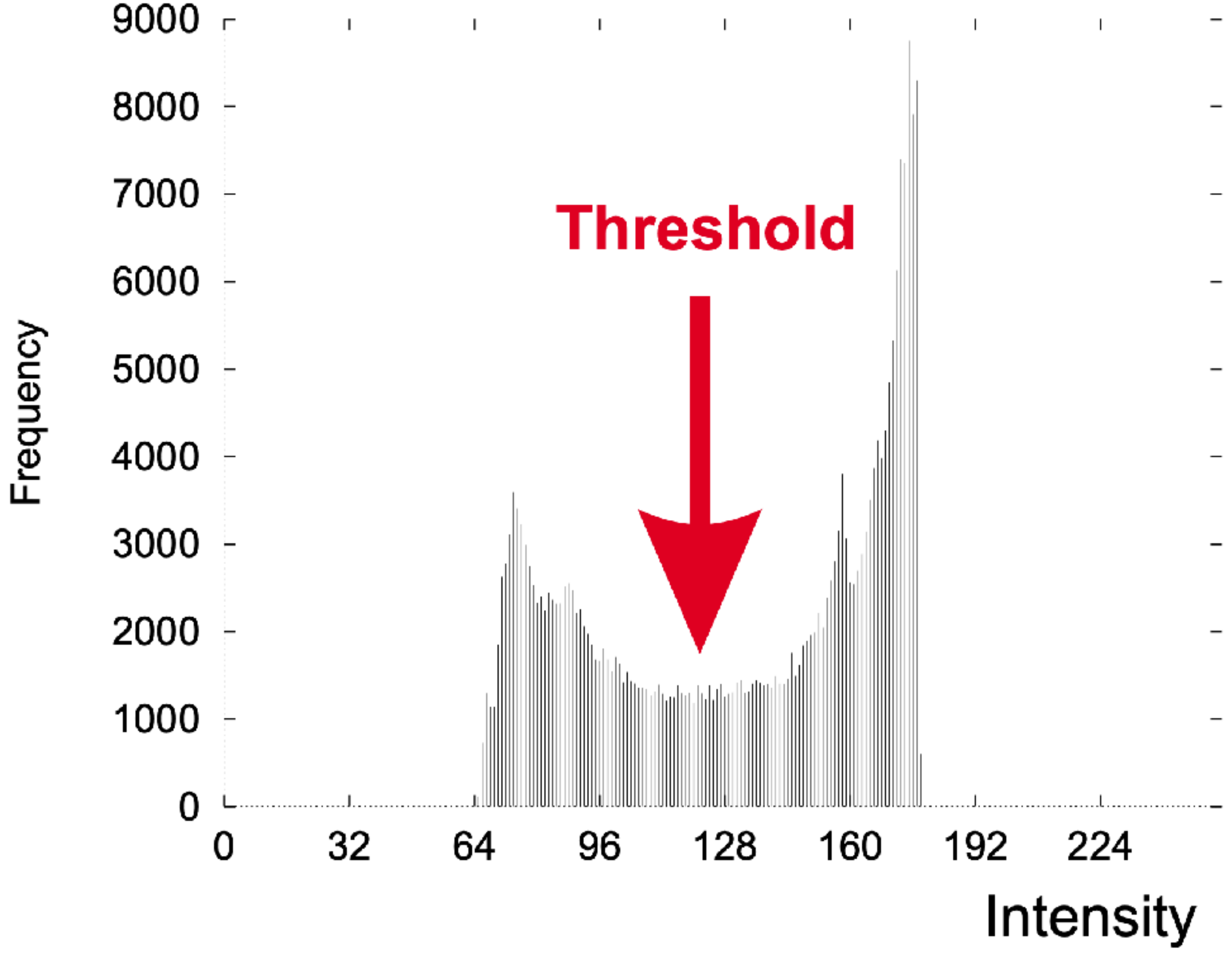

- How to handle noisy histograms?

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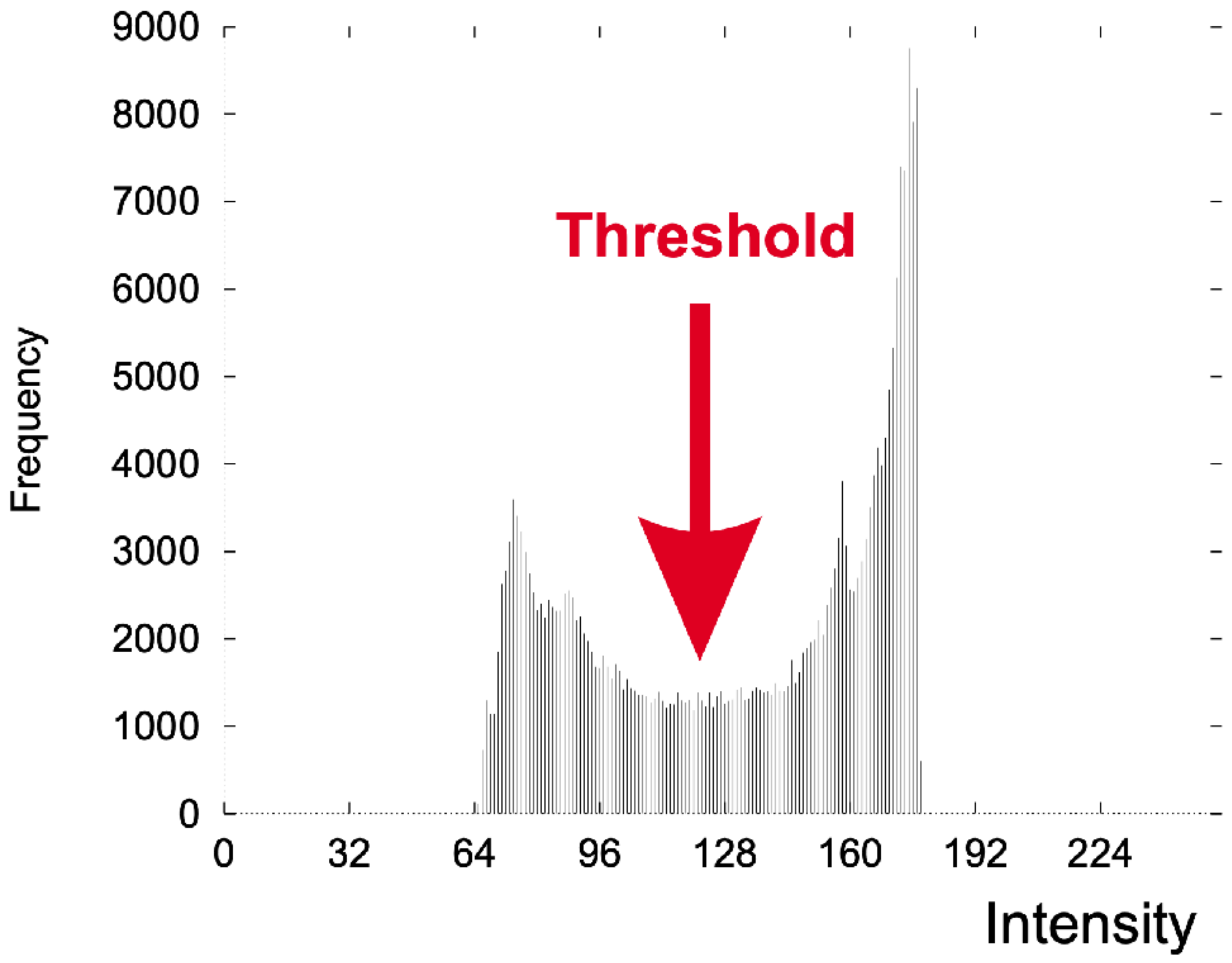

- How to handle noisy histograms?
- How to find optimal threshold(s)?

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Smoothing Histograms

- Noise in observations (e.g., noisy pixel intensities) → noisy / ragged histograms

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slide credit: Václav Hlaváč

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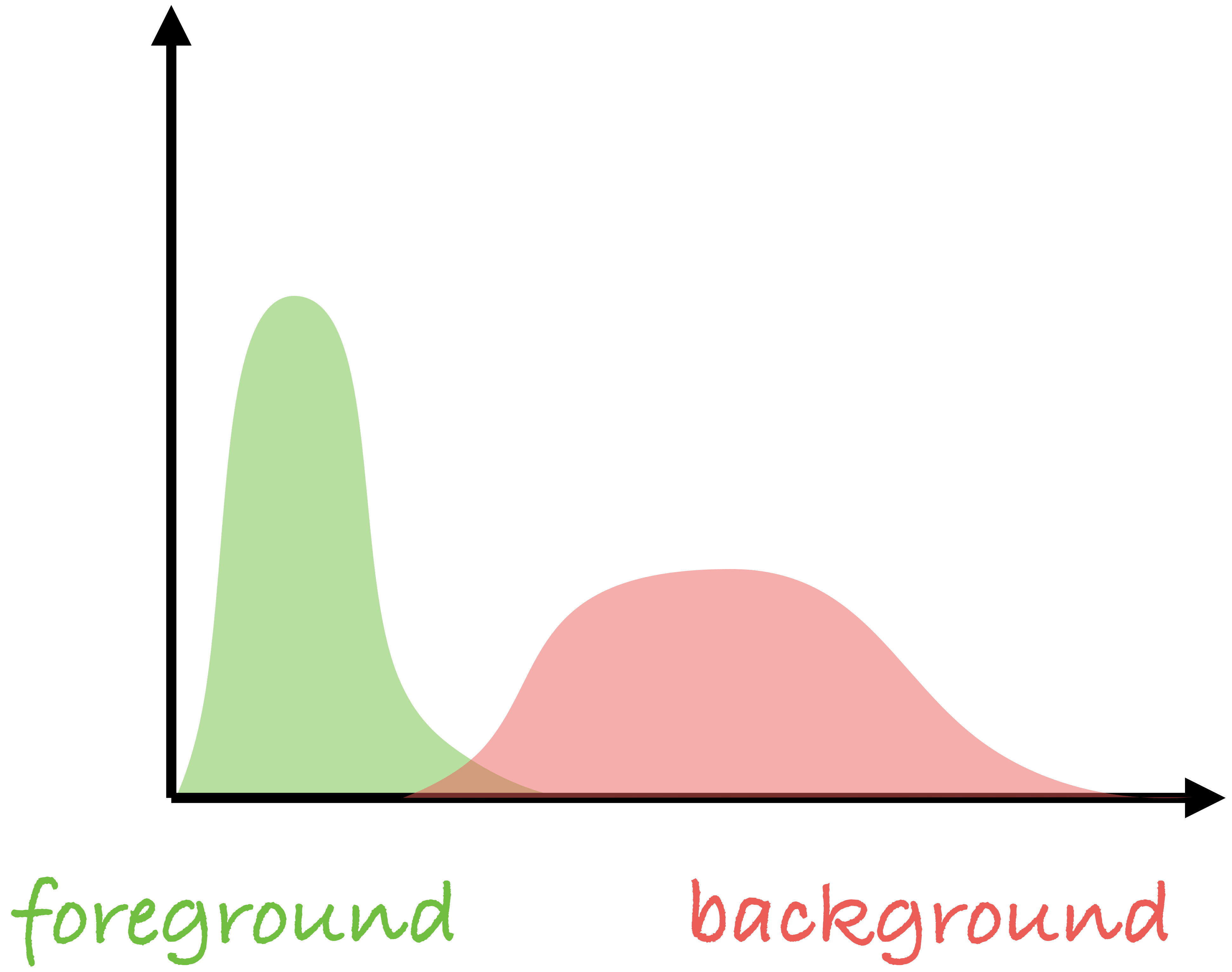
  - Input histogram $h(i)$ over intensities $i = 0, \ldots, i_{\max}$
  - New histogram $h'(i)$ after applying sliding average with window size $2K+1$

$$h(i) = \frac{1}{2K+1} \sum_{j=-K}^{K} h(i+j), \quad i = K, \ldots, i_{\max} - K$$

slide credit: Václav Hlaváč

“Optimal” Thresholding Via Mixture of Gaussians



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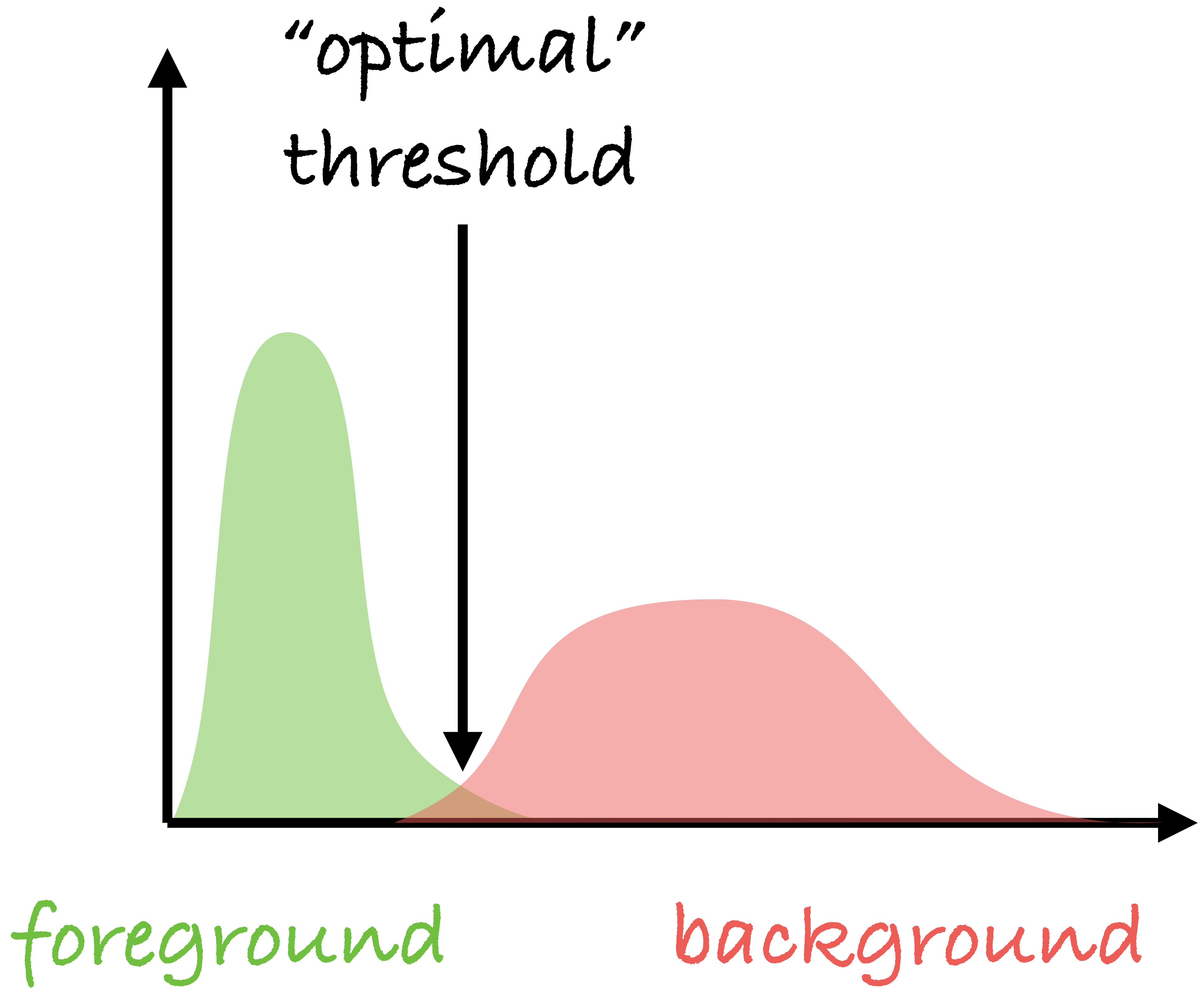

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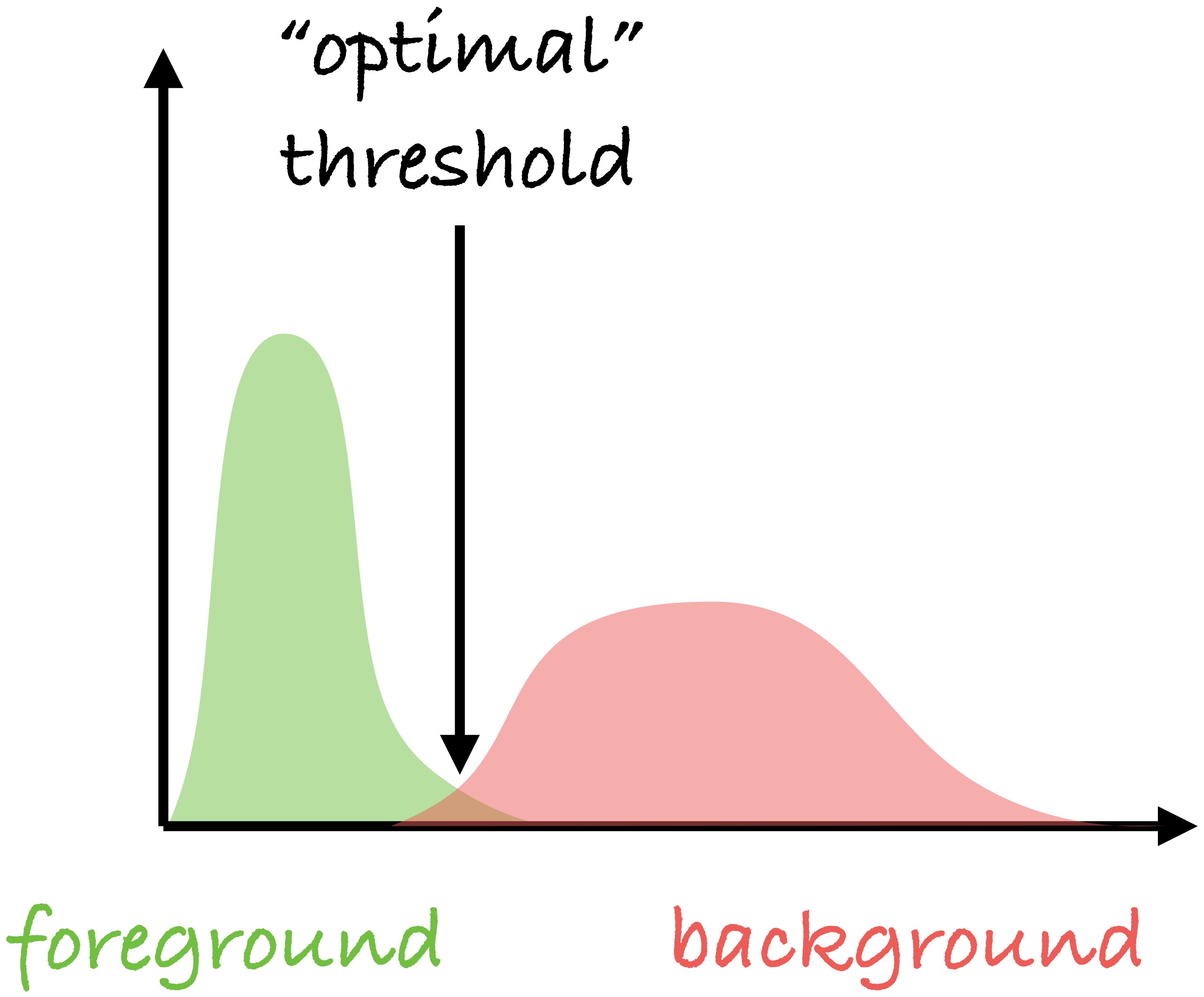

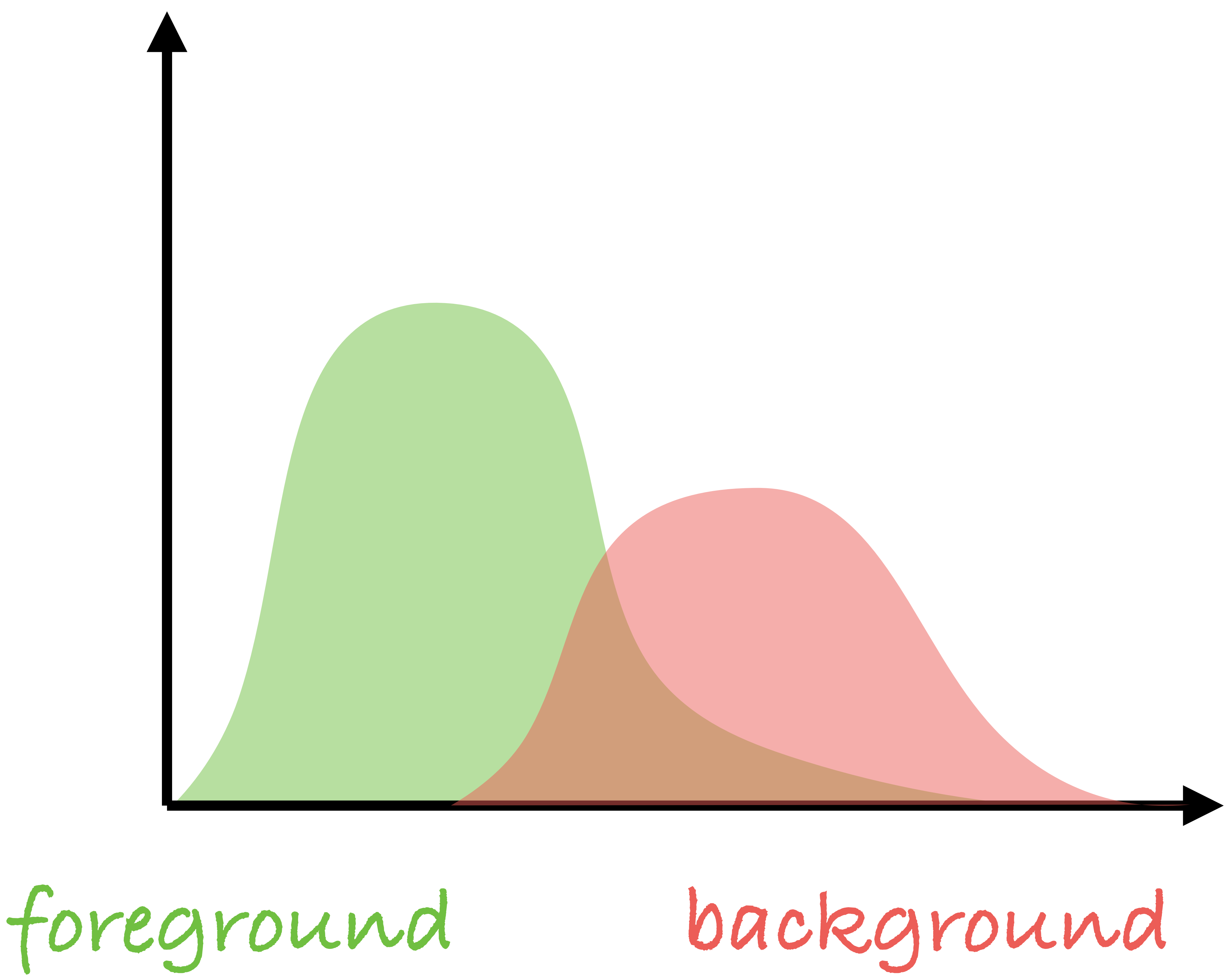

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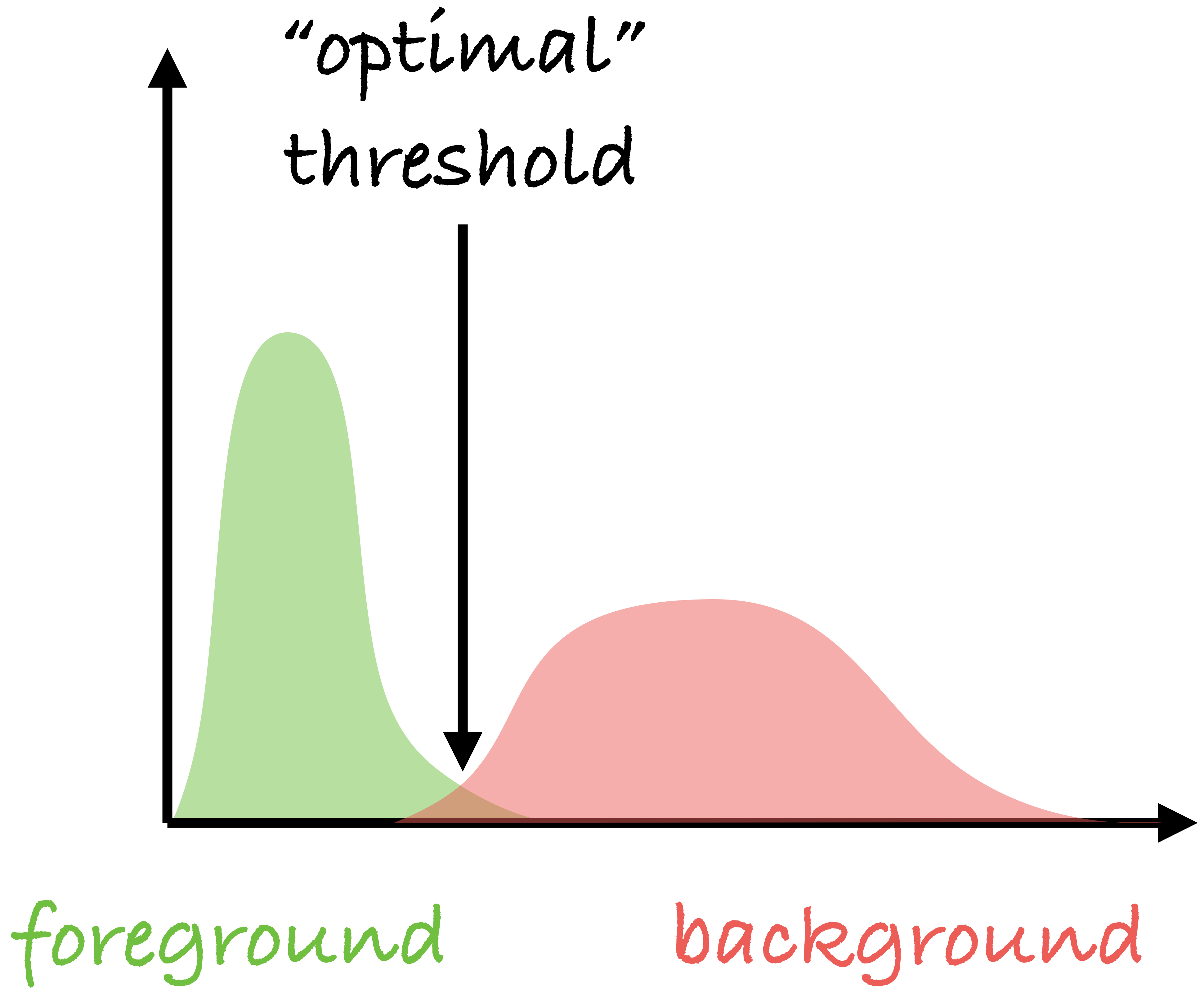

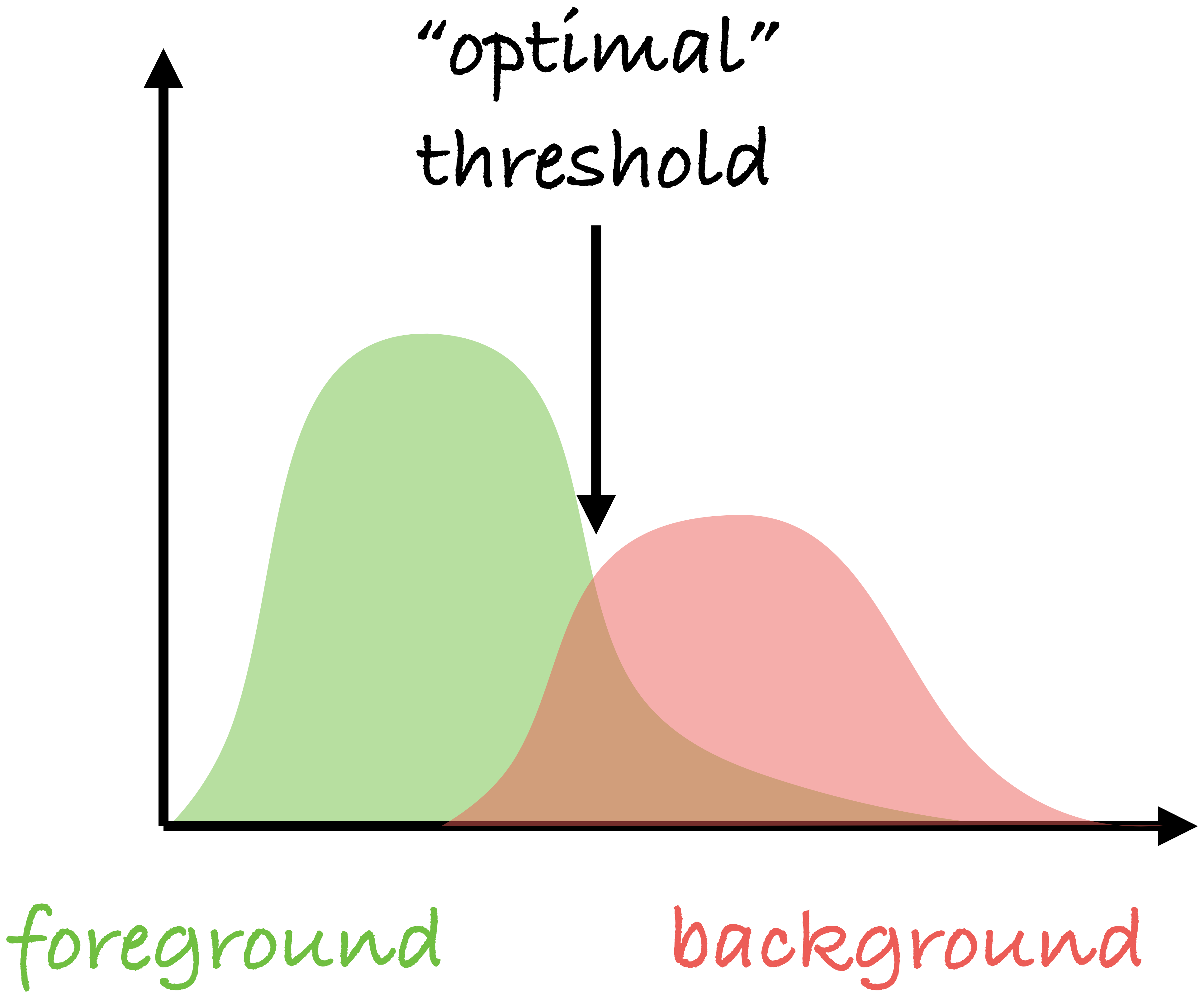

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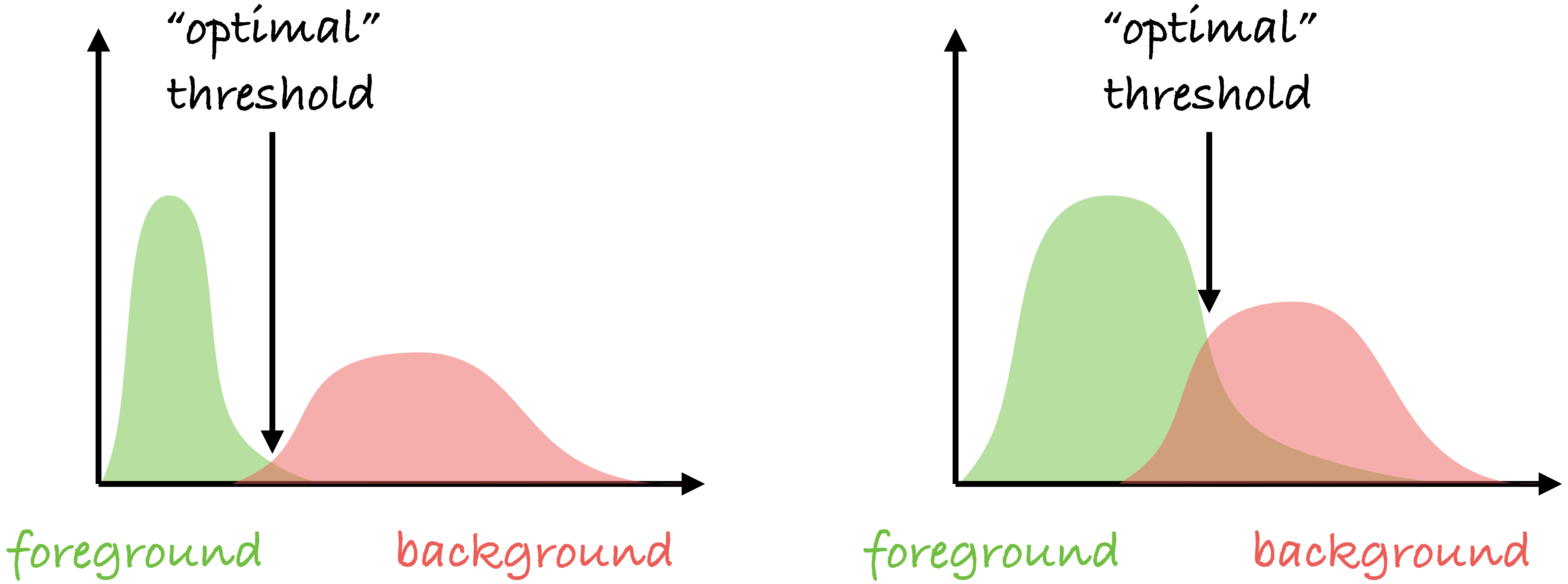

Choose thresholds based on decision boundaries:

$$p(\text{foreground} \mid x) > p(\text{background} \mid x)$$

slide credit: Václav Hlaváč

Fitting A Mixture of Gaussians

- Input: observed histogram $h(g)$

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Fitting A Mixture of Gaussians

- Input: observed histogram $h(g)$
- Estimate: approximate histogram $h_{\text{model}}(g)$ modeled by *n* Gaussians

$$h_{\text{model}}(g) = \sum_{i=1}^{n} a_i e^{-\frac{(g-\mu_i)^2}{2\sigma_i^2}}$$

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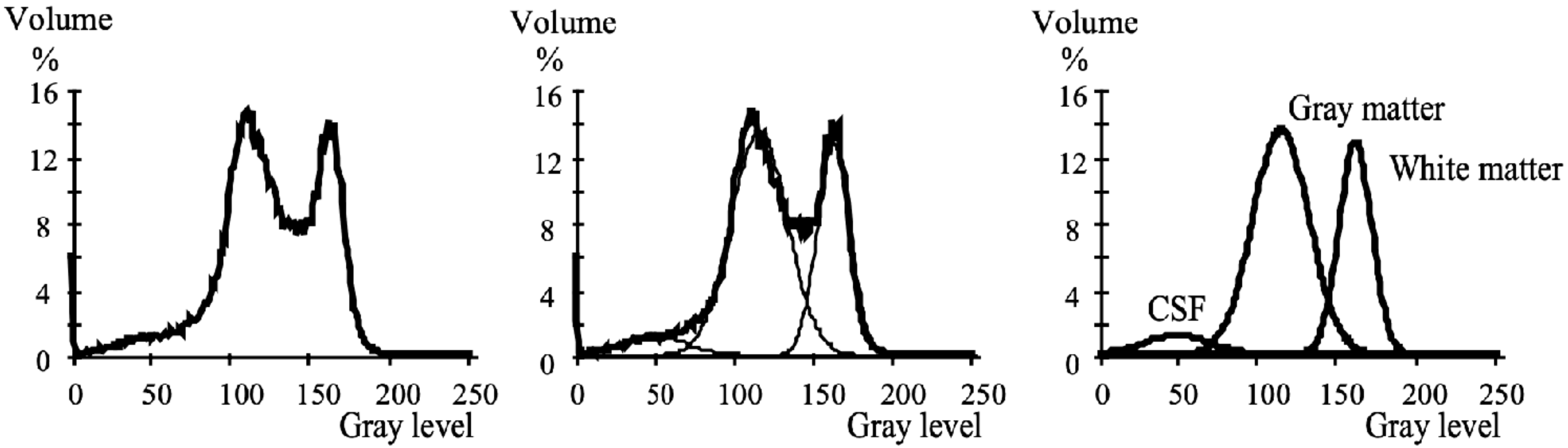
- See part on **expectation maximization**

slide credit: Václav Hlaváč

Example: Brain MRI Segmentation

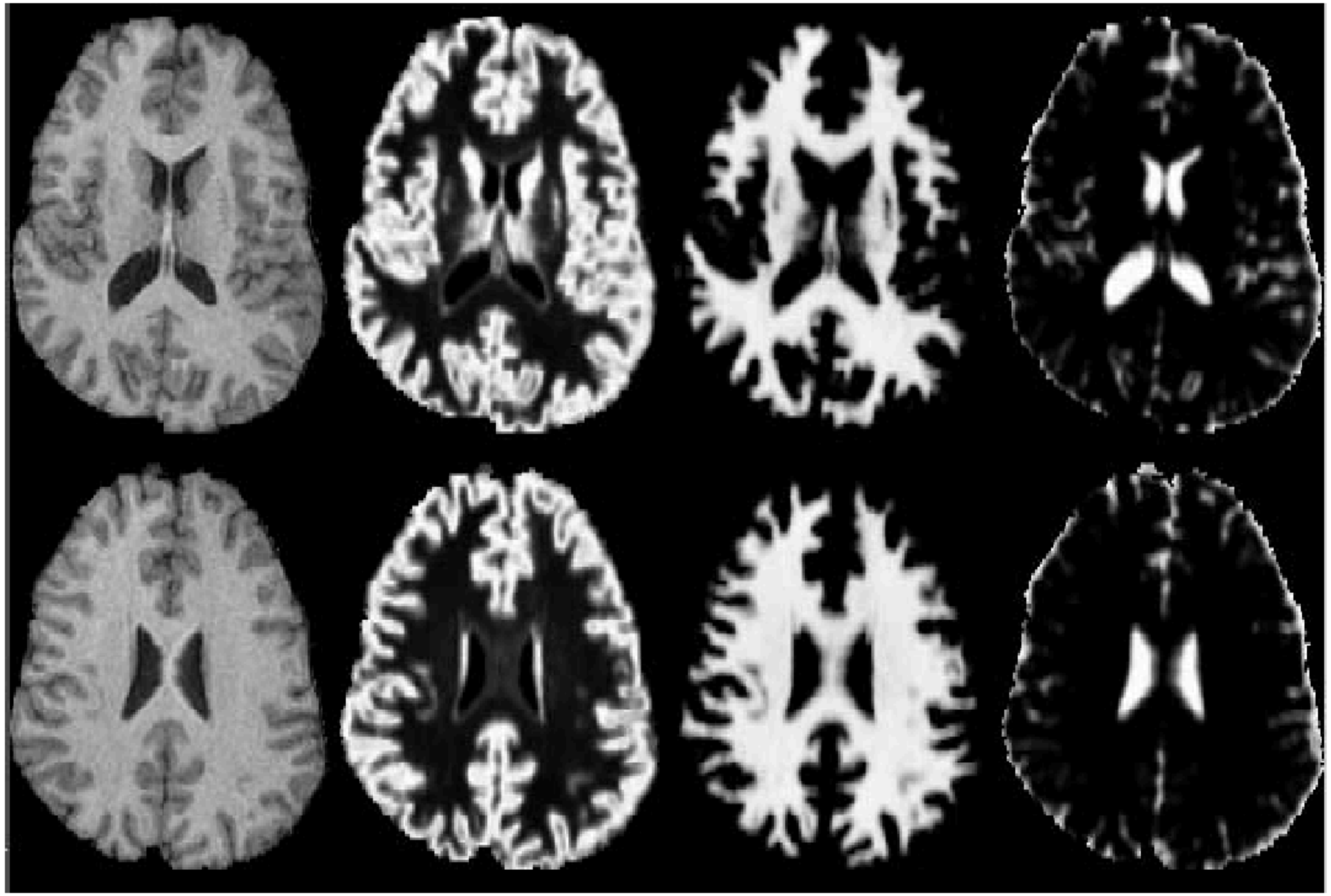
- Input: NMR images
- Classes: white matter, grey matter, celebrities-spinal fluid (CSF)



Courtesy: Milan Šonka, University of Iowa.

slide credit: Václav Hlaváč

Example: Brain MRI Segmentation

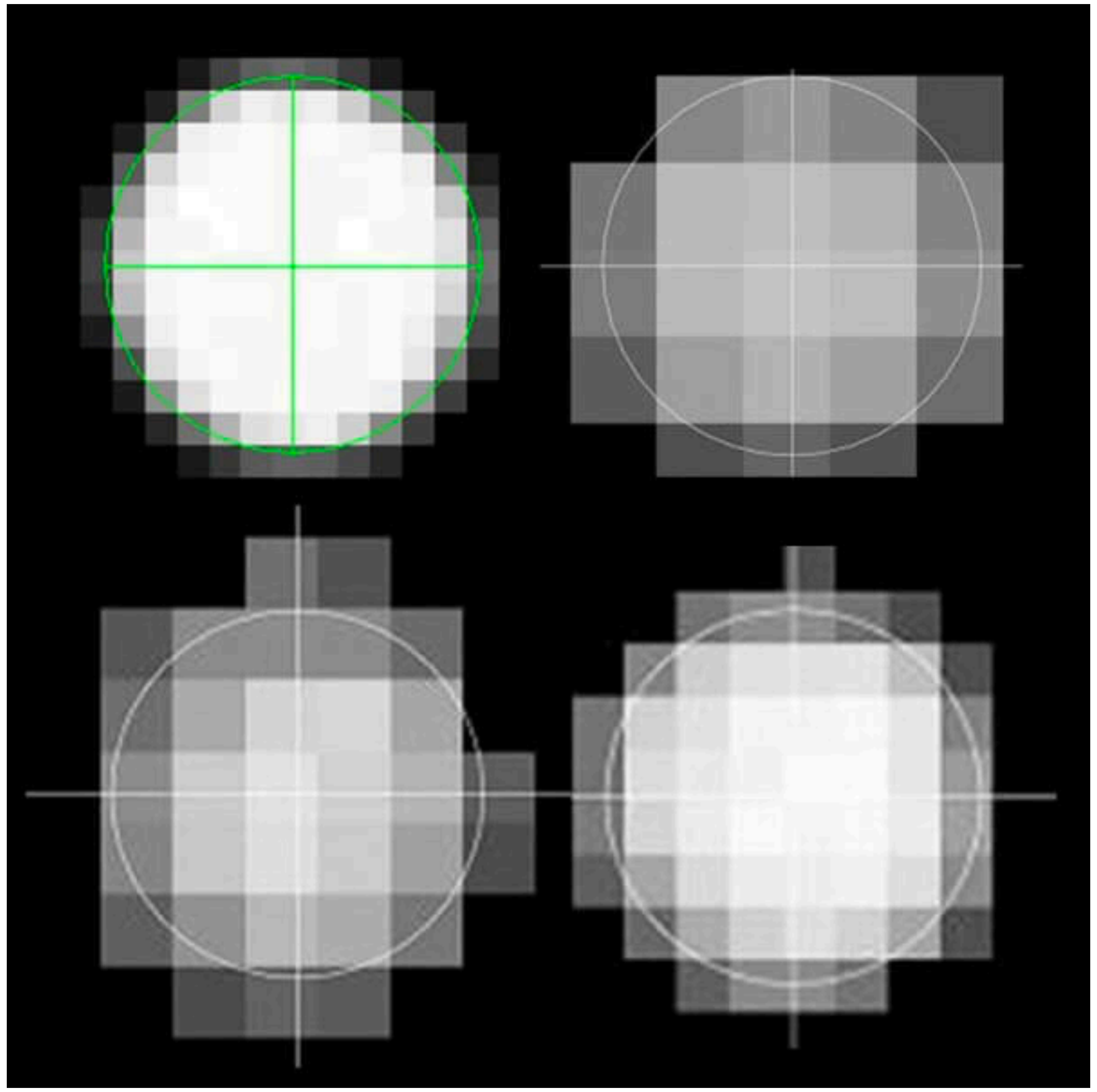


original gray matter white matter CSF

Courtesy: Milan Šonka, University of Iowa.

slide credit: Václav Hlaváč

Isn’t This Outdated?



Depending on application, we can ensure that thresholding works well!

images taken from Vicon website

Isn't This Outdated?

Compute representations where thresholding works

images from [Lianos, Schönberger, Pollefeys, Sattler, VSO: Visual Semantic Odometry]

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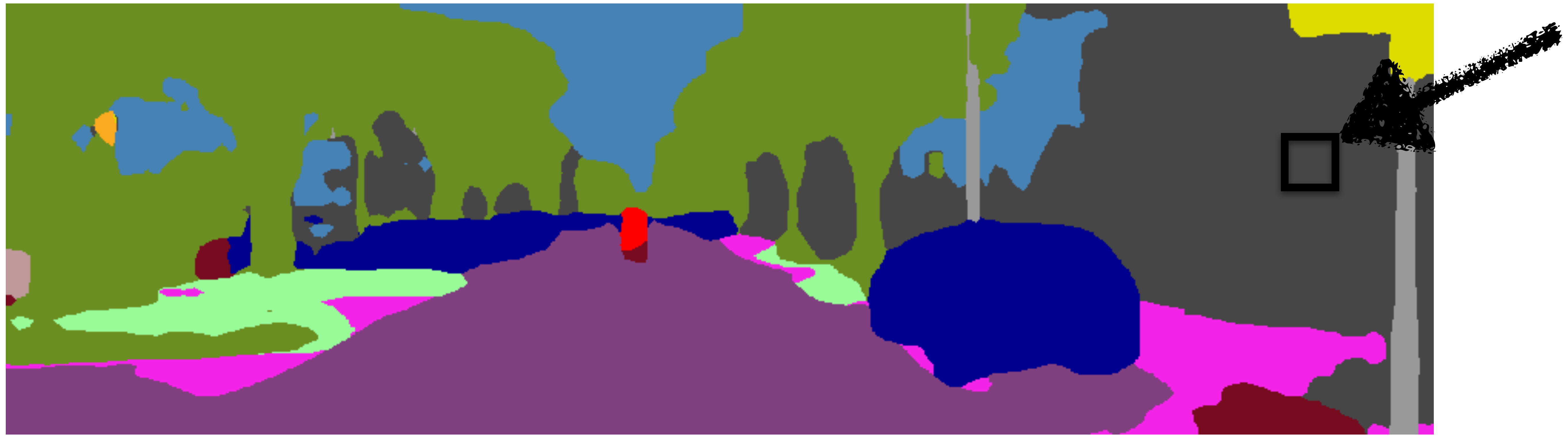
Compute representations where thresholding works



set of probabilities per pixel x

$p(x = \text{car}) = \ldots$

$p(x = \text{building}) = \ldots$

$p(x = \text{road}) = \ldots$

$\vdots$

images from [Lianos, Schönberger, Pollefeys, Sattler, VSO: Visual Semantic Odometry]

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Compute representations where thresholding works

set of probabilities per pixel x

$p(x = \text{car}) = \ldots$

$p(x = \text{building}) = \ldots$

$p(x = \text{road}) = \ldots$

$\vdots$

result of thresholding on $p(x = \text{car})$

images from [Lianos, Schönberger, Pollefeys, Sattler, VSO: Visual Semantic Odometry]

Lecture Overview

simple & heuristic

complex & principled

- A simple approach to segmentation: **(intensity) thresholding**
- Segmentation based on spatial coherence: **edge-based segmentation**, **region growing**
- Segmentation as a **clustering** problem: **k-means clustering**, **mean-shift clustering**
- Segmentation as a statistical (unsupervised) learning problem: **expectation maximization (EM) algorithm**
- Next lecture: **graph-based segmentation**, **supervised learning with neural networks** (if time and interest)

slide credit: Václav Hlaváč

Thresholding Summarized

- **Pro**: very easy to implement

slide credit: Václav Hlaváč

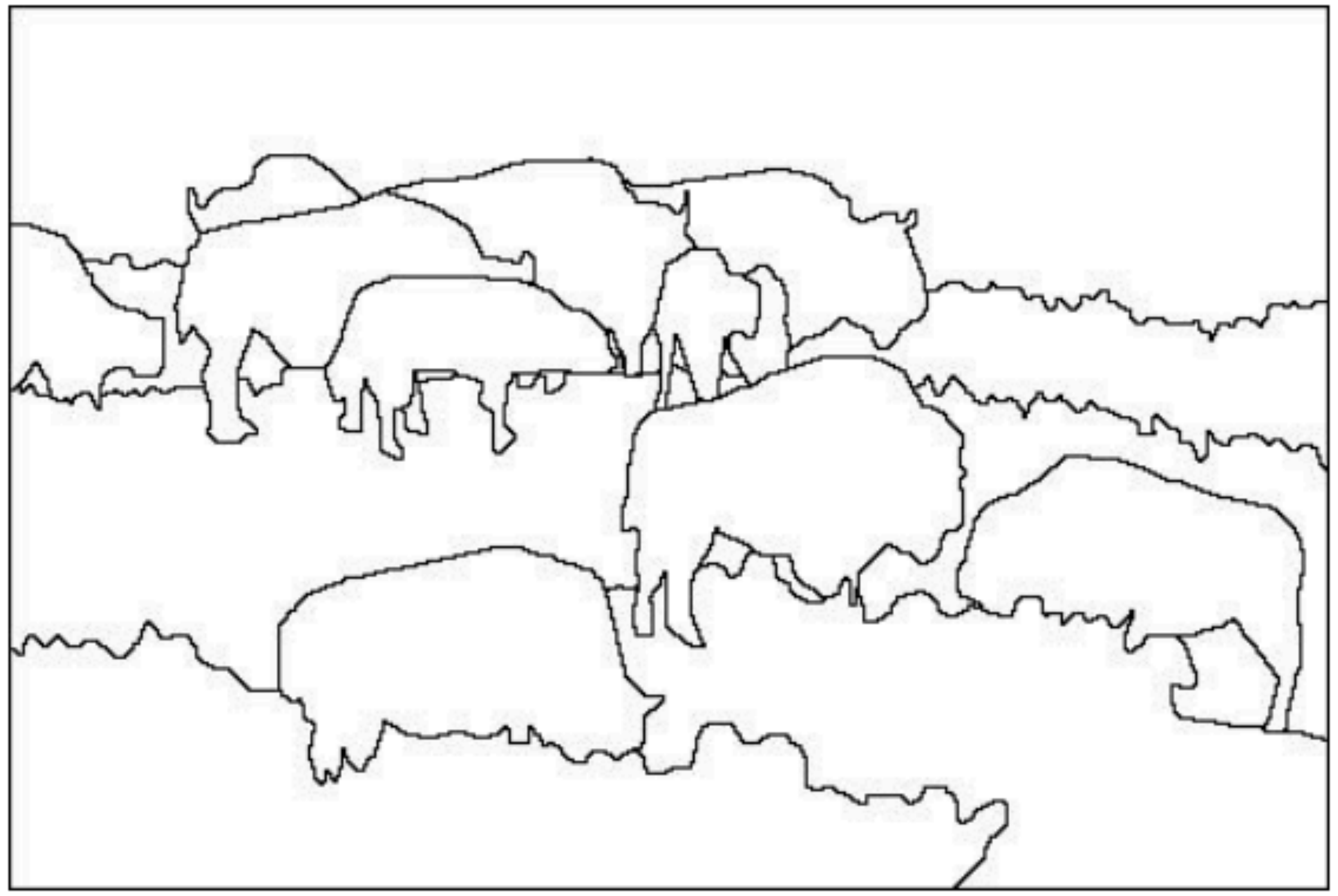
Thresholding Summarized

- **Pro**: very easy to implement
- **Pro**: very easy to parallelize

slide credit: Václav Hlaváč

Thresholding Summarized

- **Pro**: very easy to implement
- **Pro**: very easy to parallelize
- **Con**: we are ignoring that we are looking for **regions** of pixels that belong together → ignoring **spatial consistency**



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Thresholding Summarized

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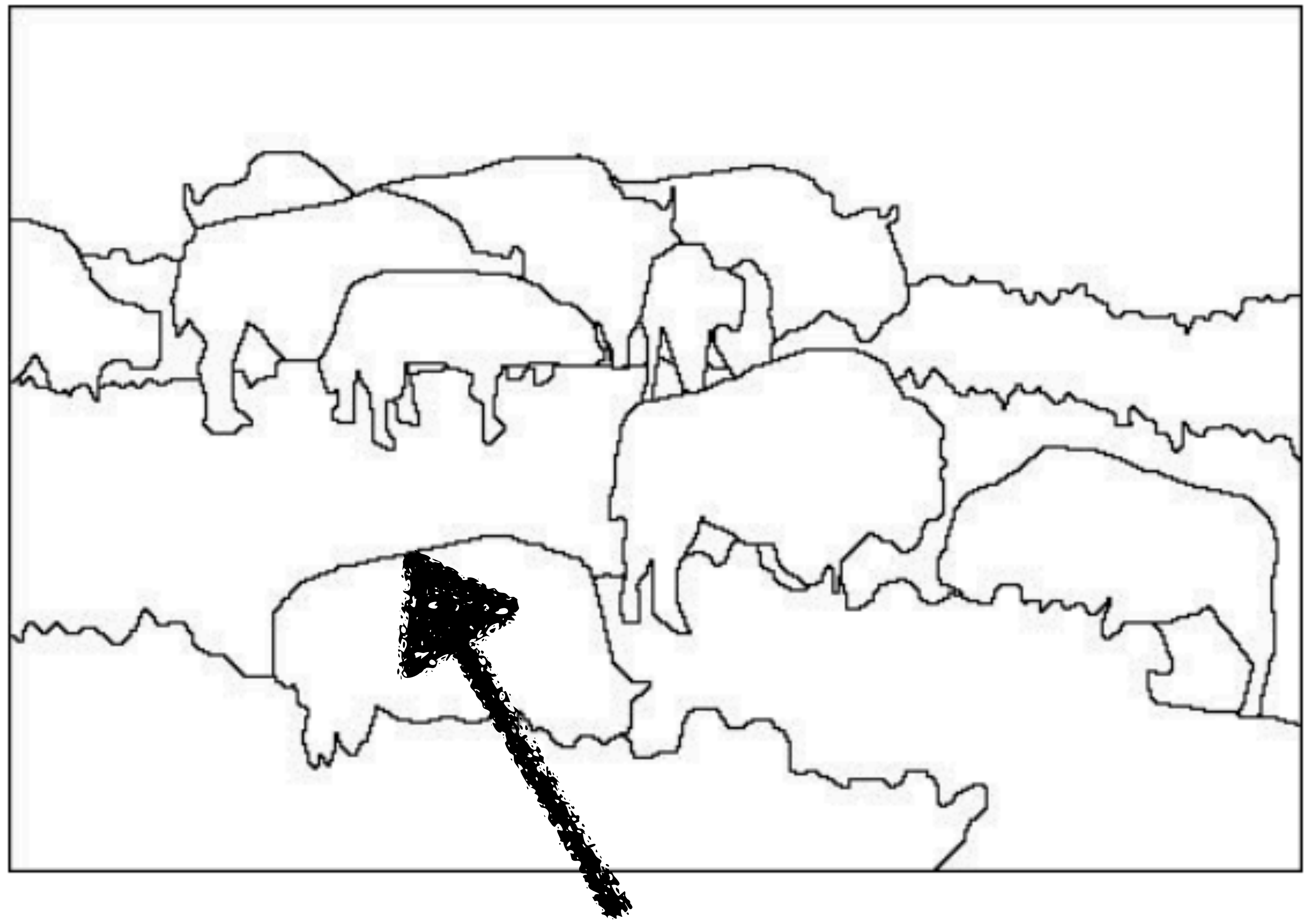

edges = region boundaries

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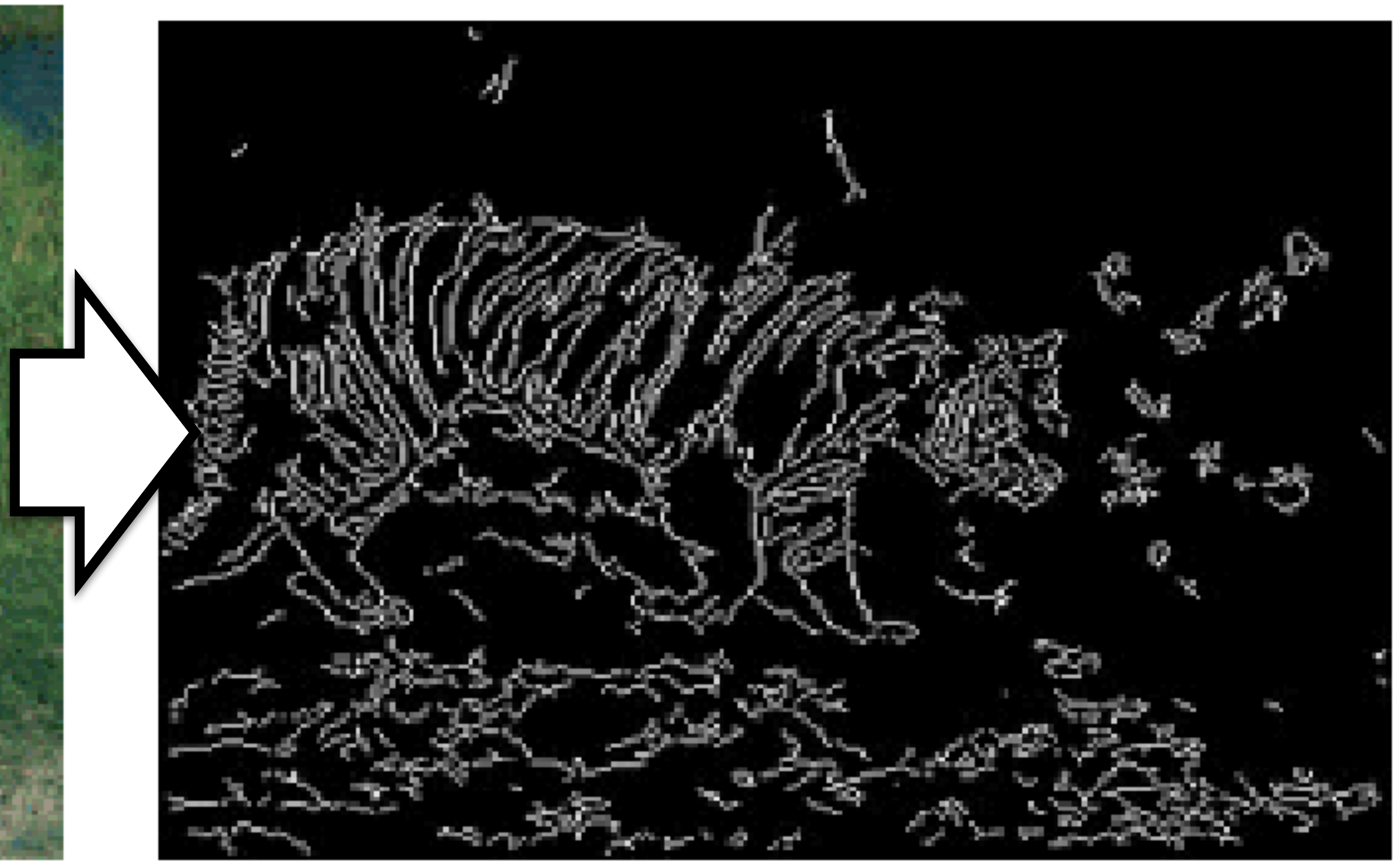
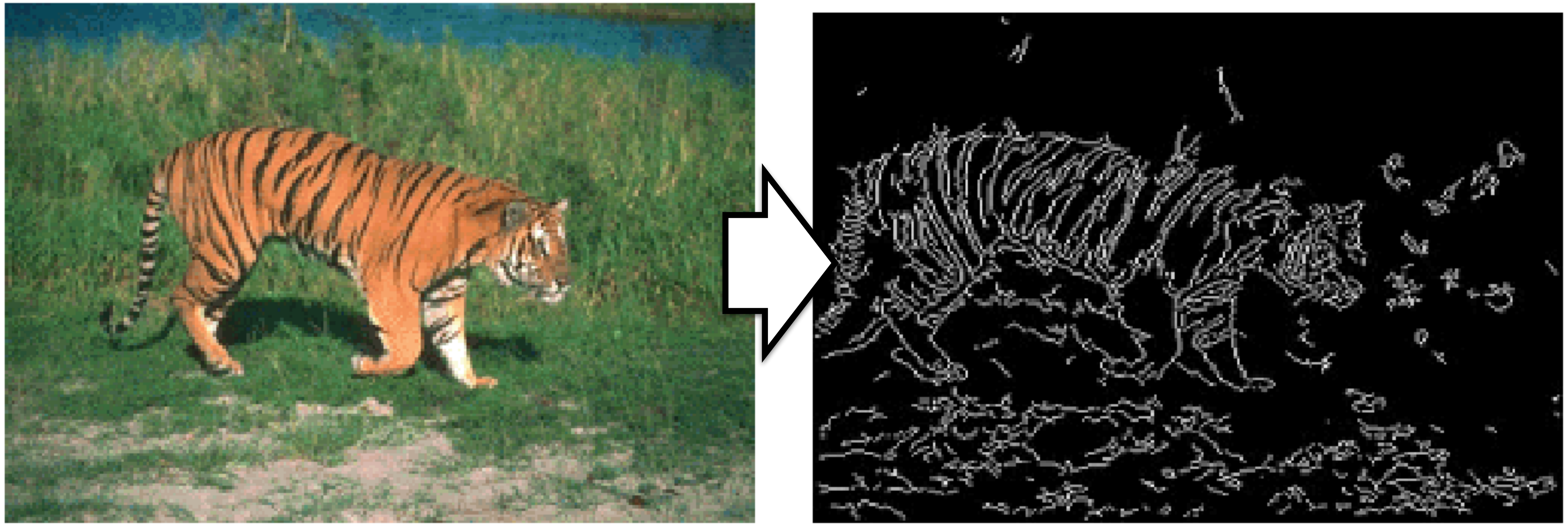
Edge-Based Image Segmentation



- **Edgels**: significant edges from edge detector (e.g., Canny)

slide credit: Václav Hlaváč

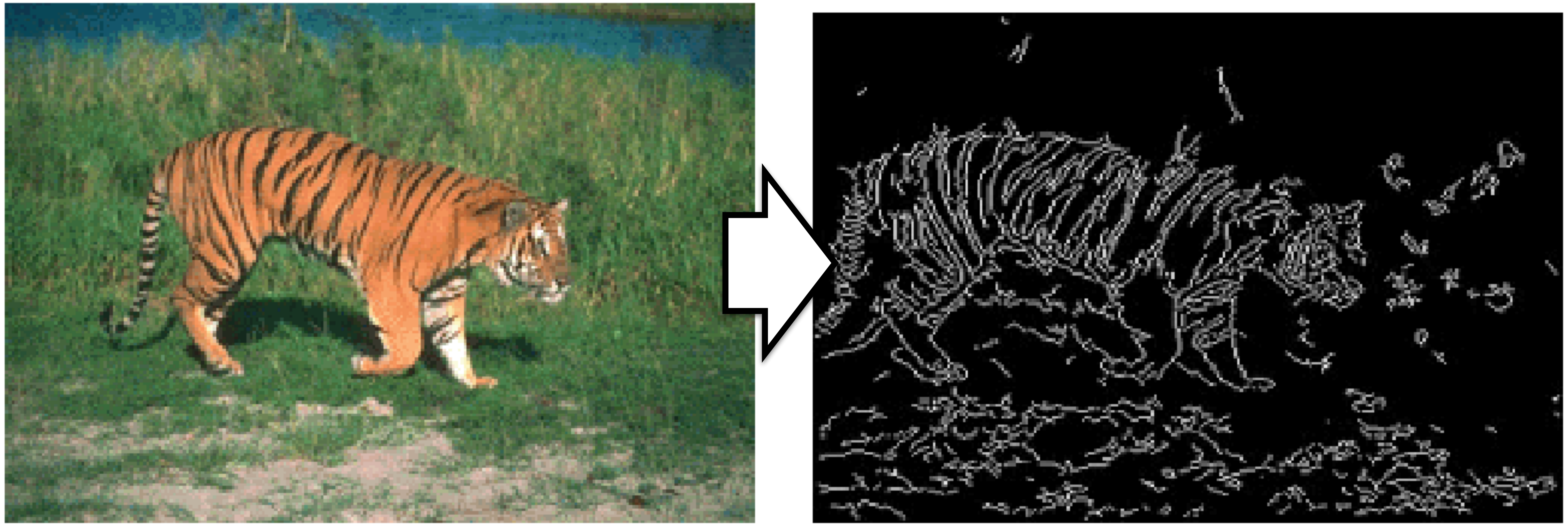
Edge-Based Image Segmentation



- **Edgels**: significant edges from edge detector (e.g., Canny)
- **Processing**: link edges, followed by relaxation, voting, dynamic programming, etc. to obtain region boundaries

slide credit: Václav Hlaváč

Edge-Based Image Segmentation



- **Edgels**: significant edges from edge detector (e.g., Canny)
- **Processing**: link edges, followed by relaxation, voting, dynamic programming, etc. to obtain region boundaries
- Leads to **partial segmentation** that requires post-processing, some regions might not be segmented at all

slide credit: Václav Hlaváč

Edge-Based Image Segmentation

original image

edge image
(enhanced for display)

slide credit: Václav Hlaváč

Edge-Based Image Segmentation

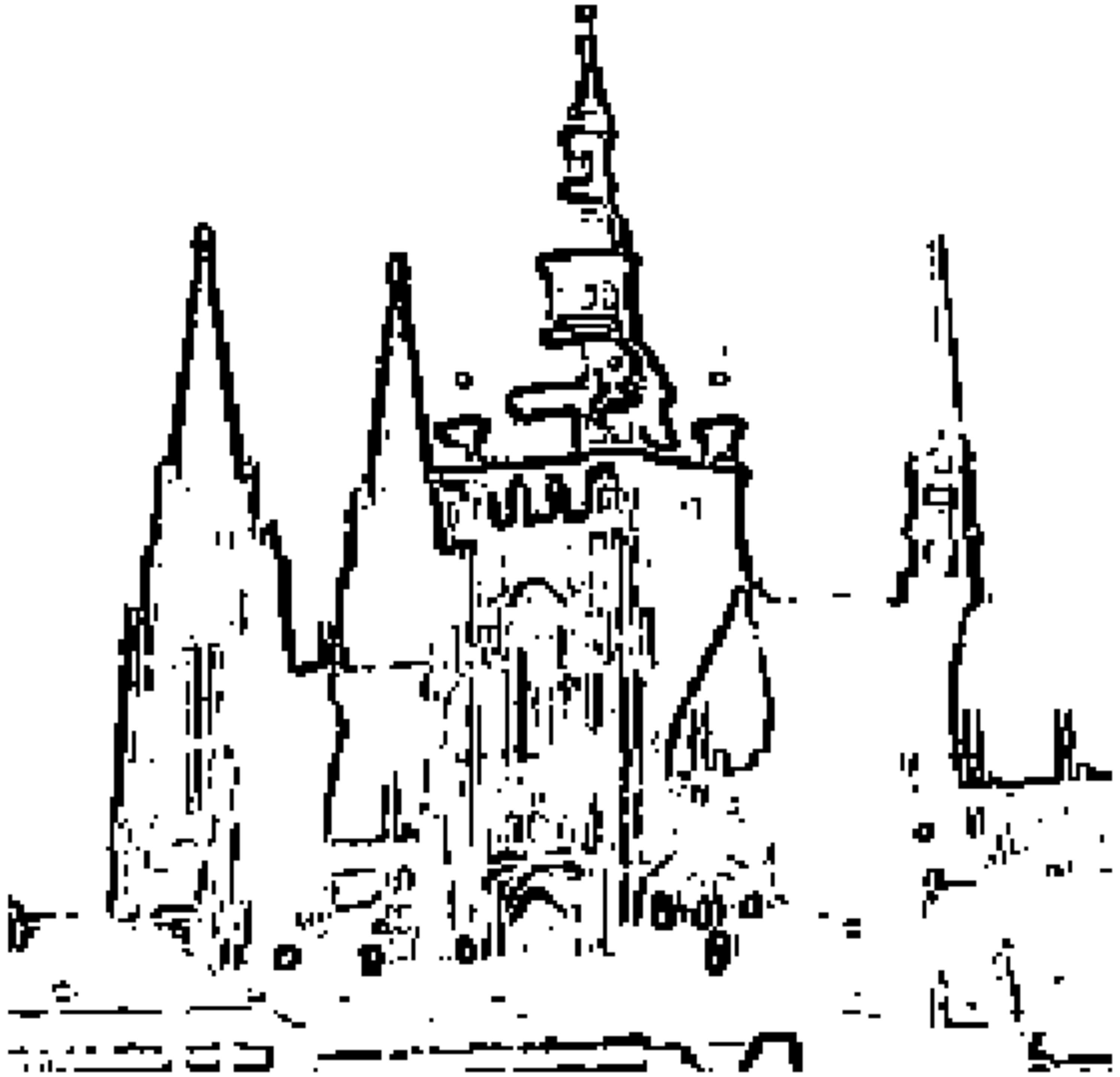
original image

edge image
(enhanced for display)

How to get from edge(l)s to region boundaries?

slide credit: Václav Hlaváč

Thresholding Edge Responses



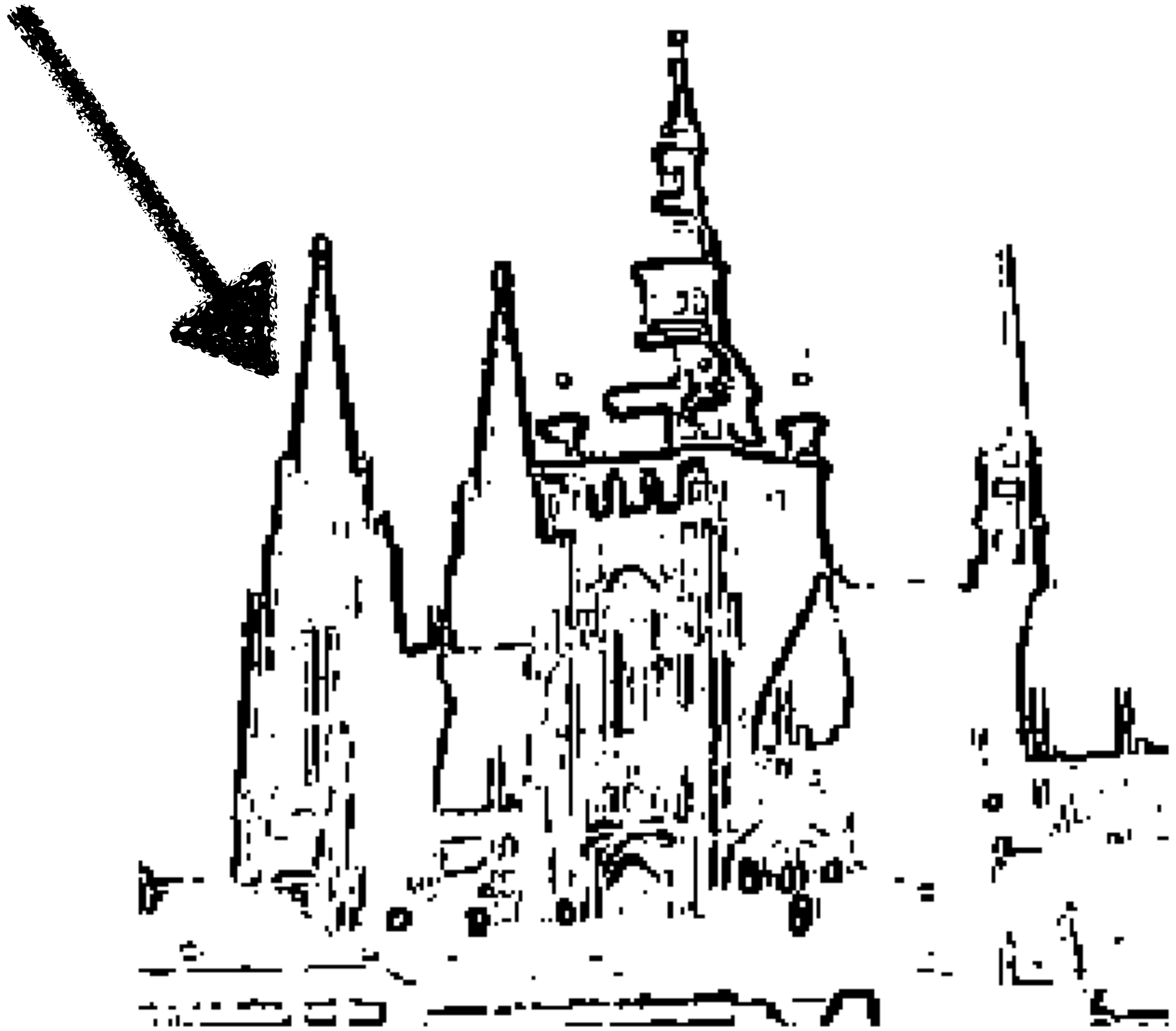
threshold 30

threshold 10 (too small)

slide credit: Václav Hlaváč

Thresholding Edge Responses

thick edges, multiple pixels wide



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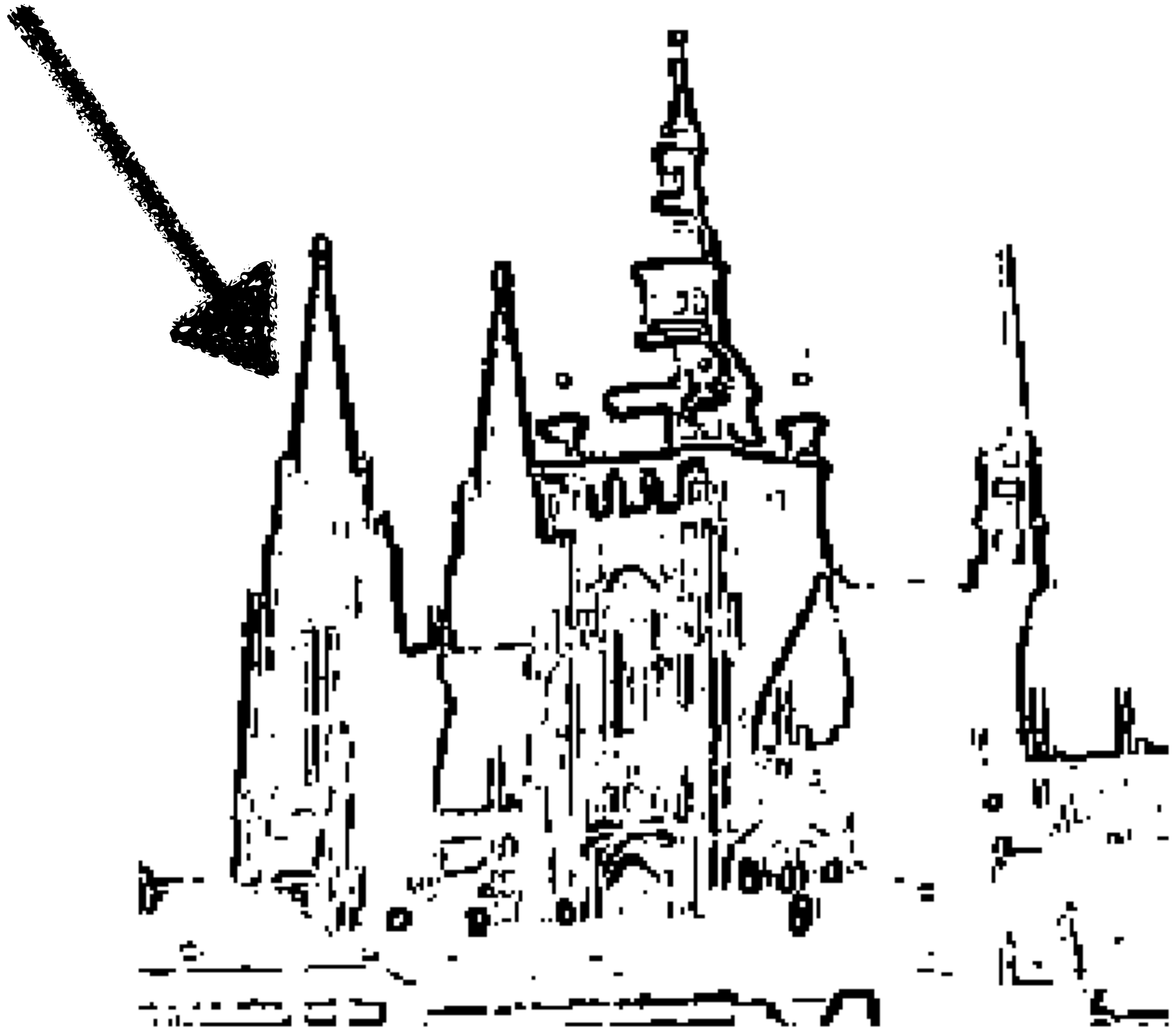


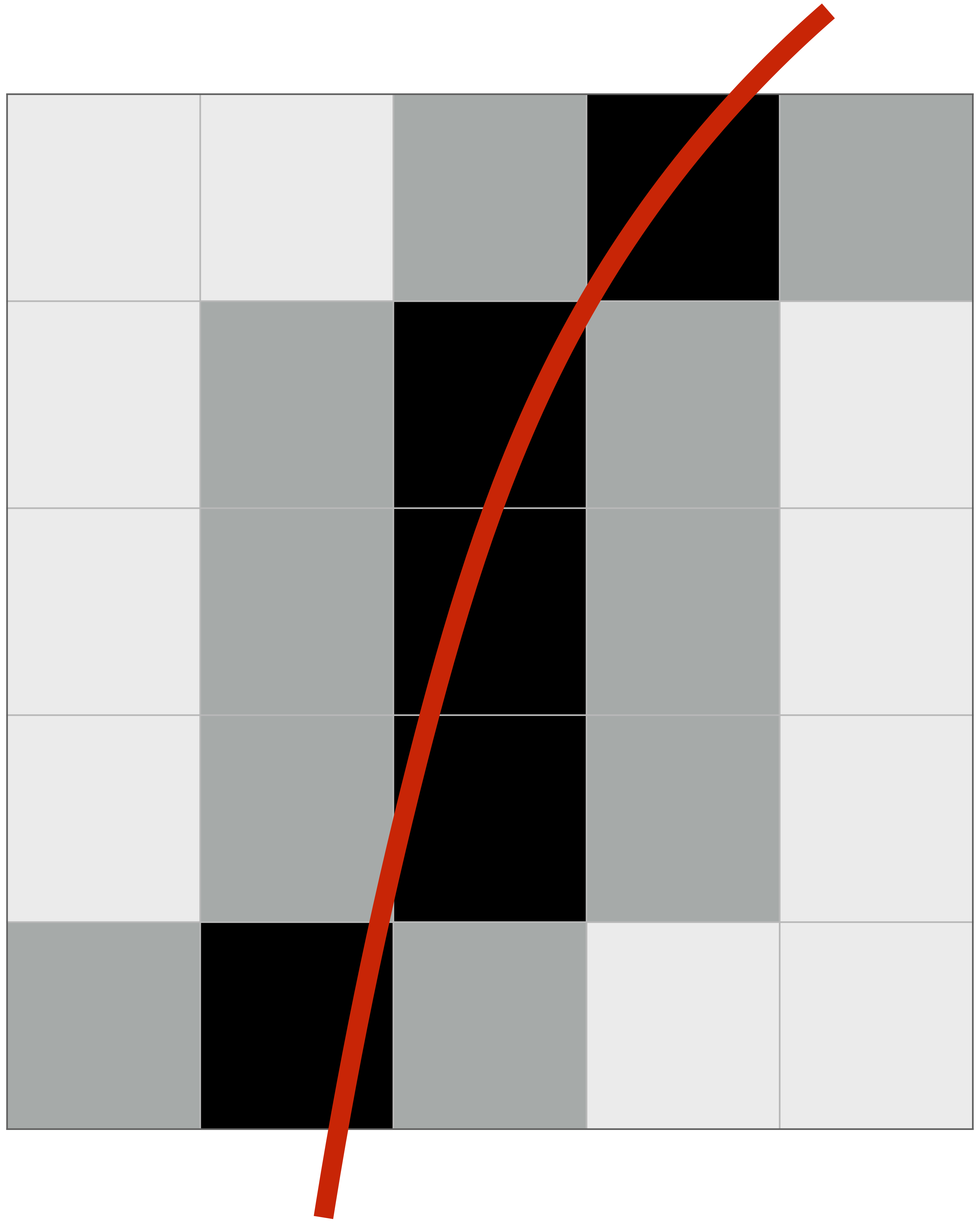
threshold 30

threshold 10 (too small)

How to get thin edges?

slide credit: Václav Hlaváč

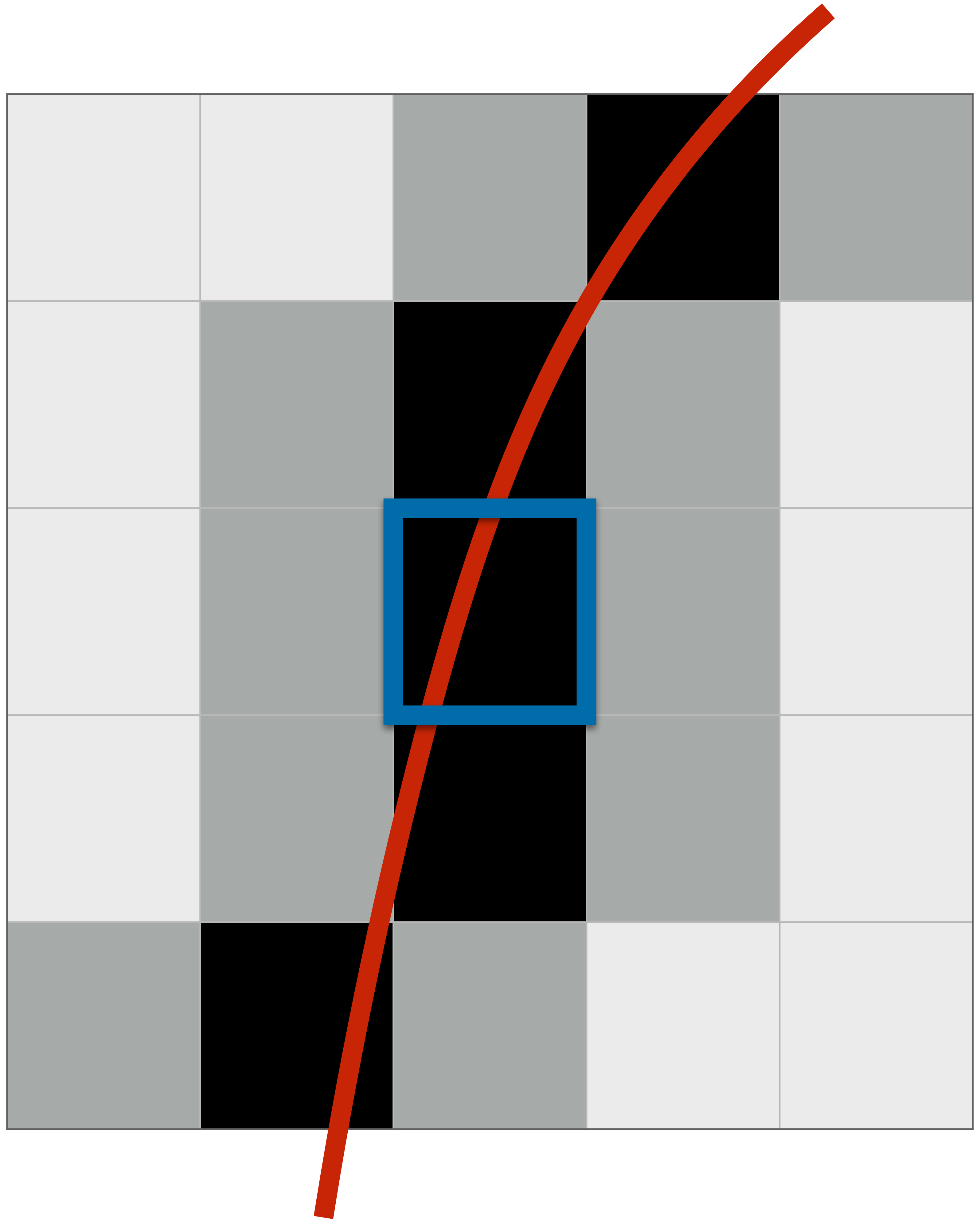
Non-Maximum Suppression For Thin Edges



- Compare pixel intensity to neighbors
- Keep only local maxima

slide credit: Václav Hlaváč

Non-Maximum Suppression For Thin Edges



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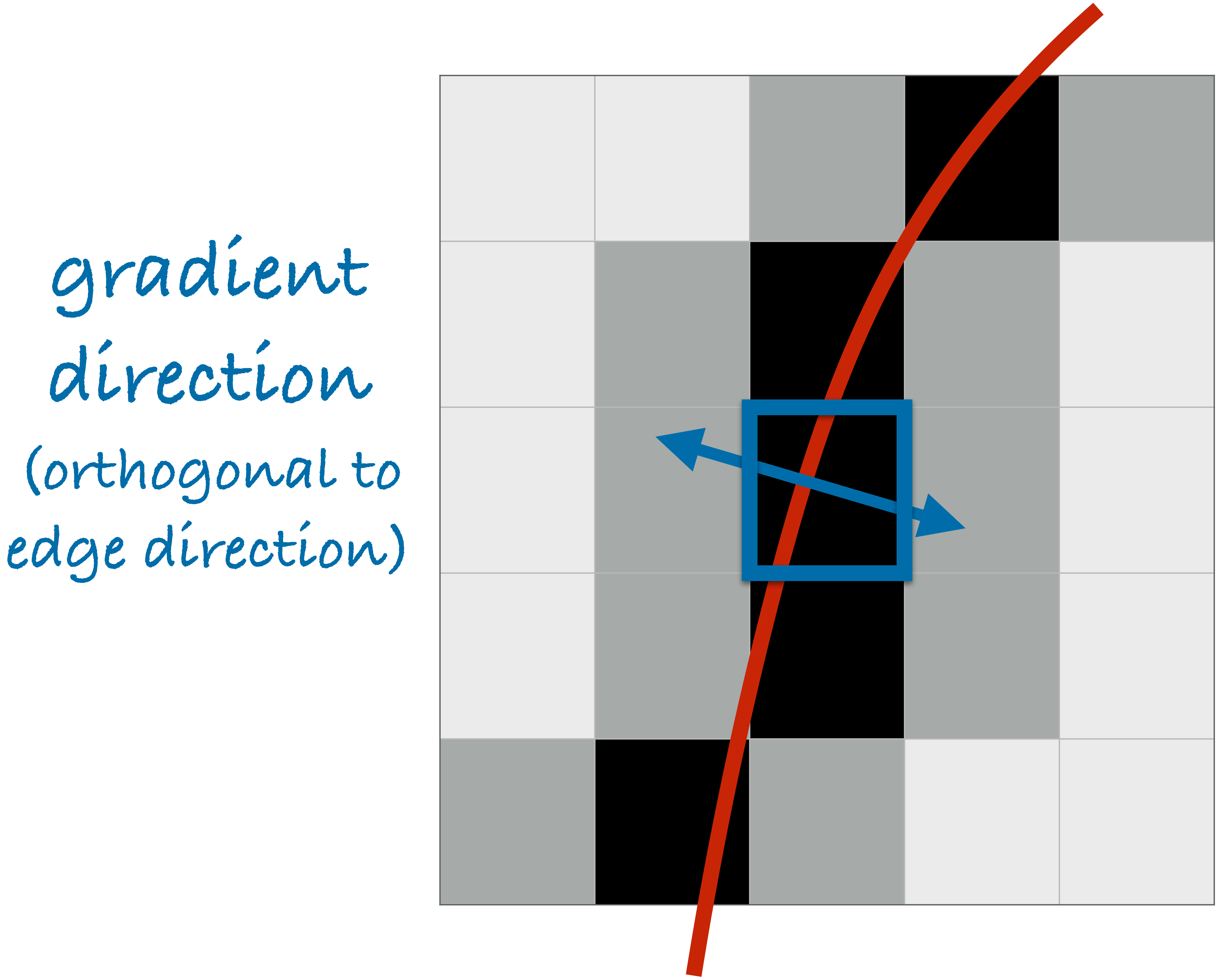

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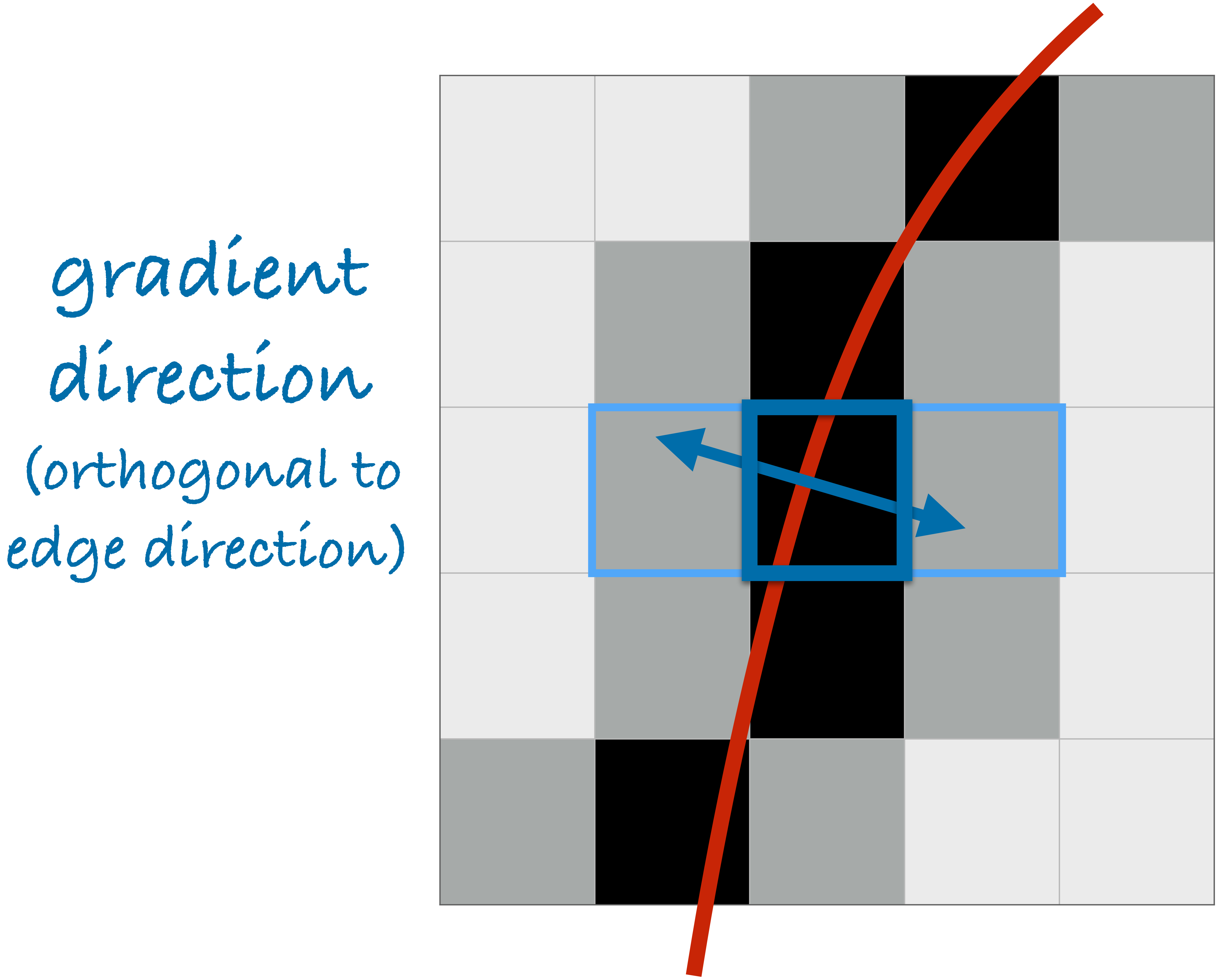

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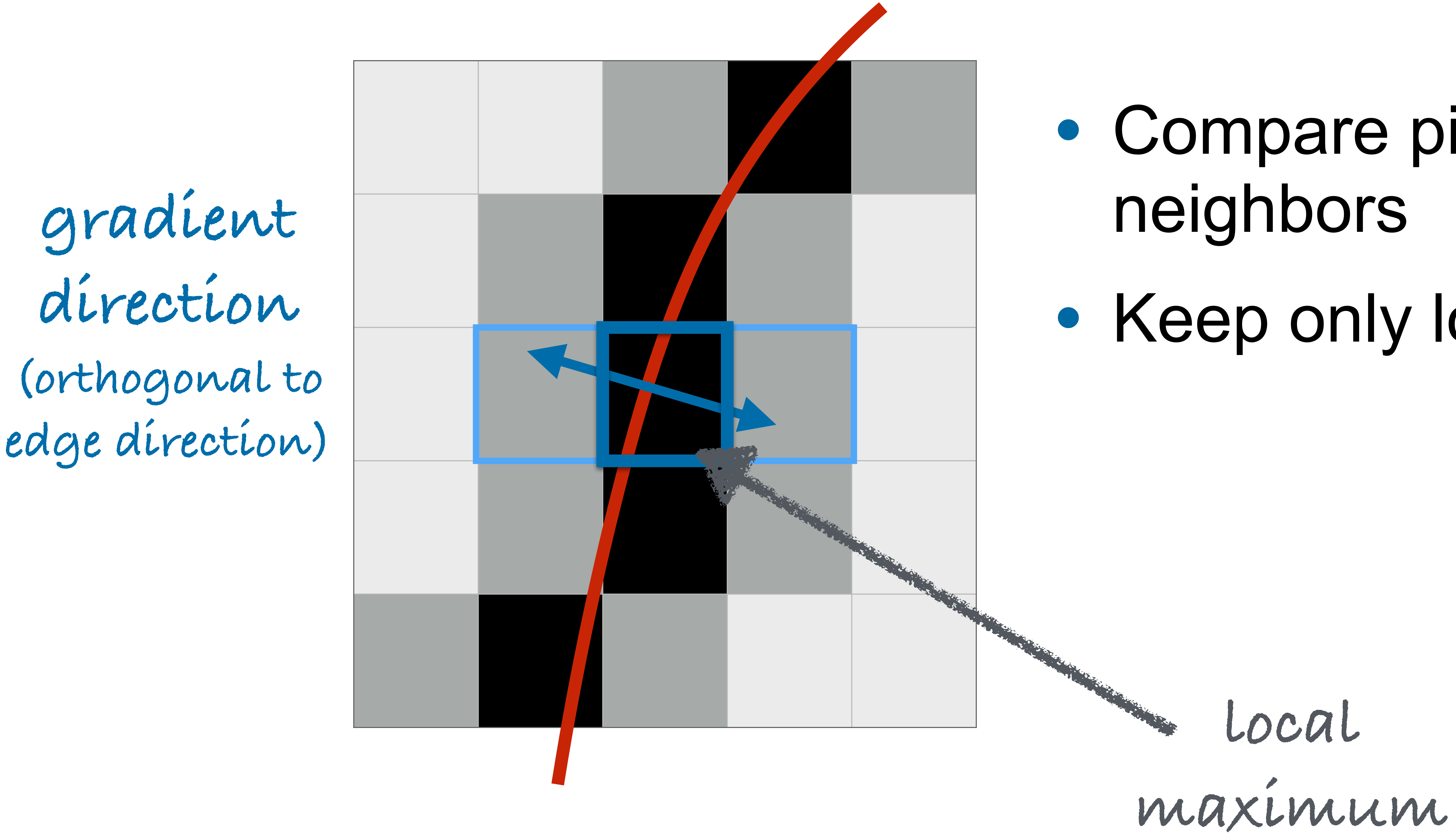

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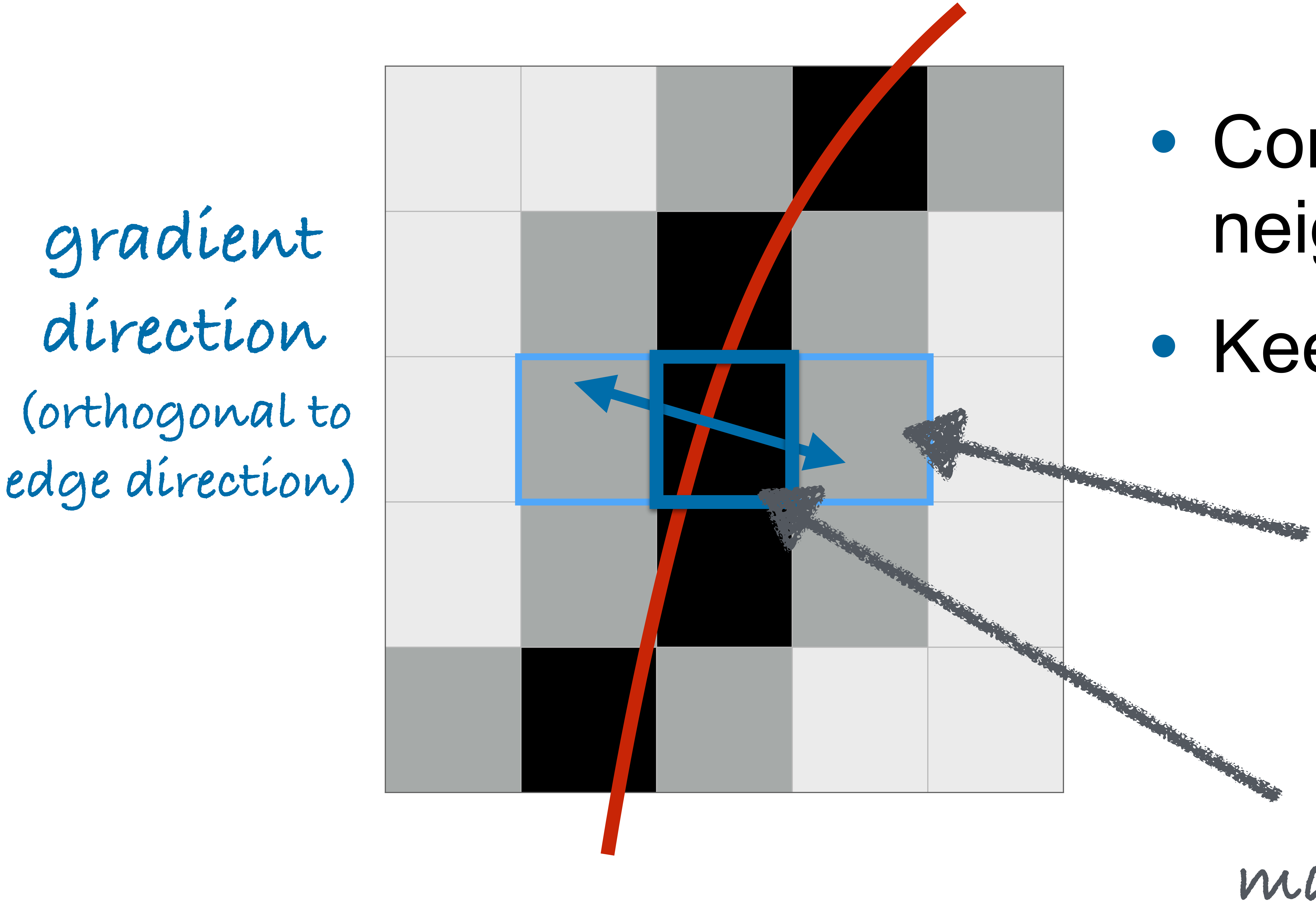

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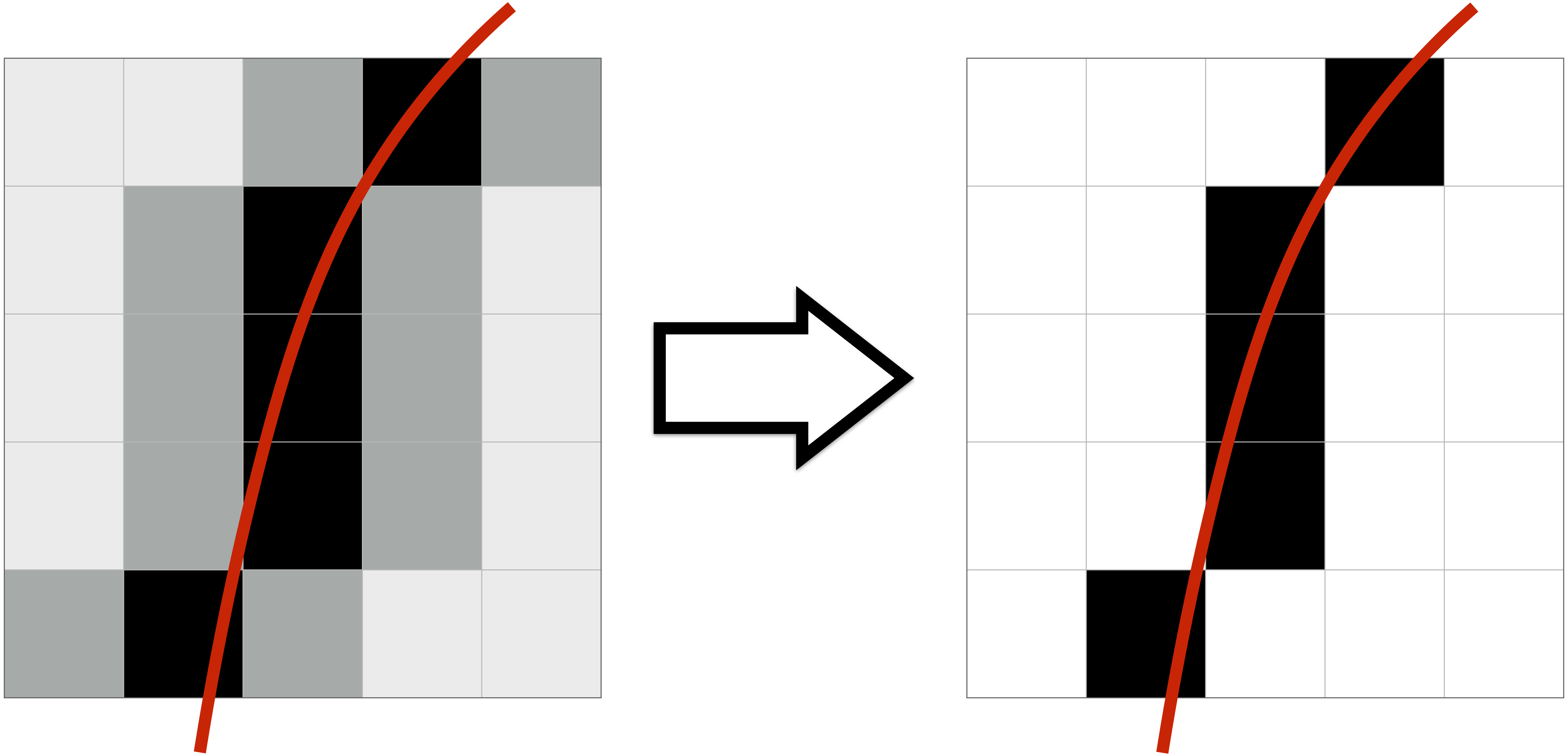

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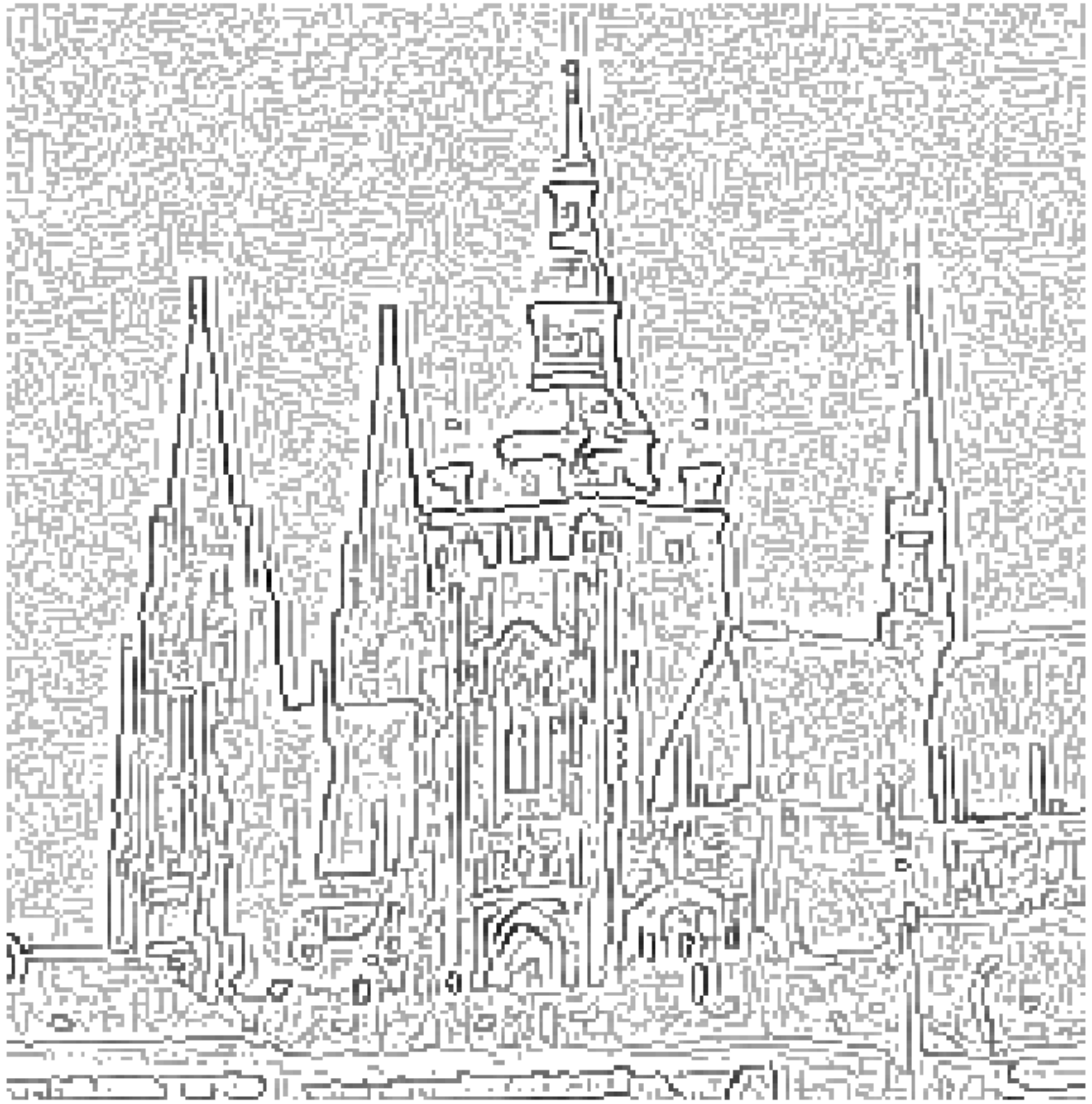
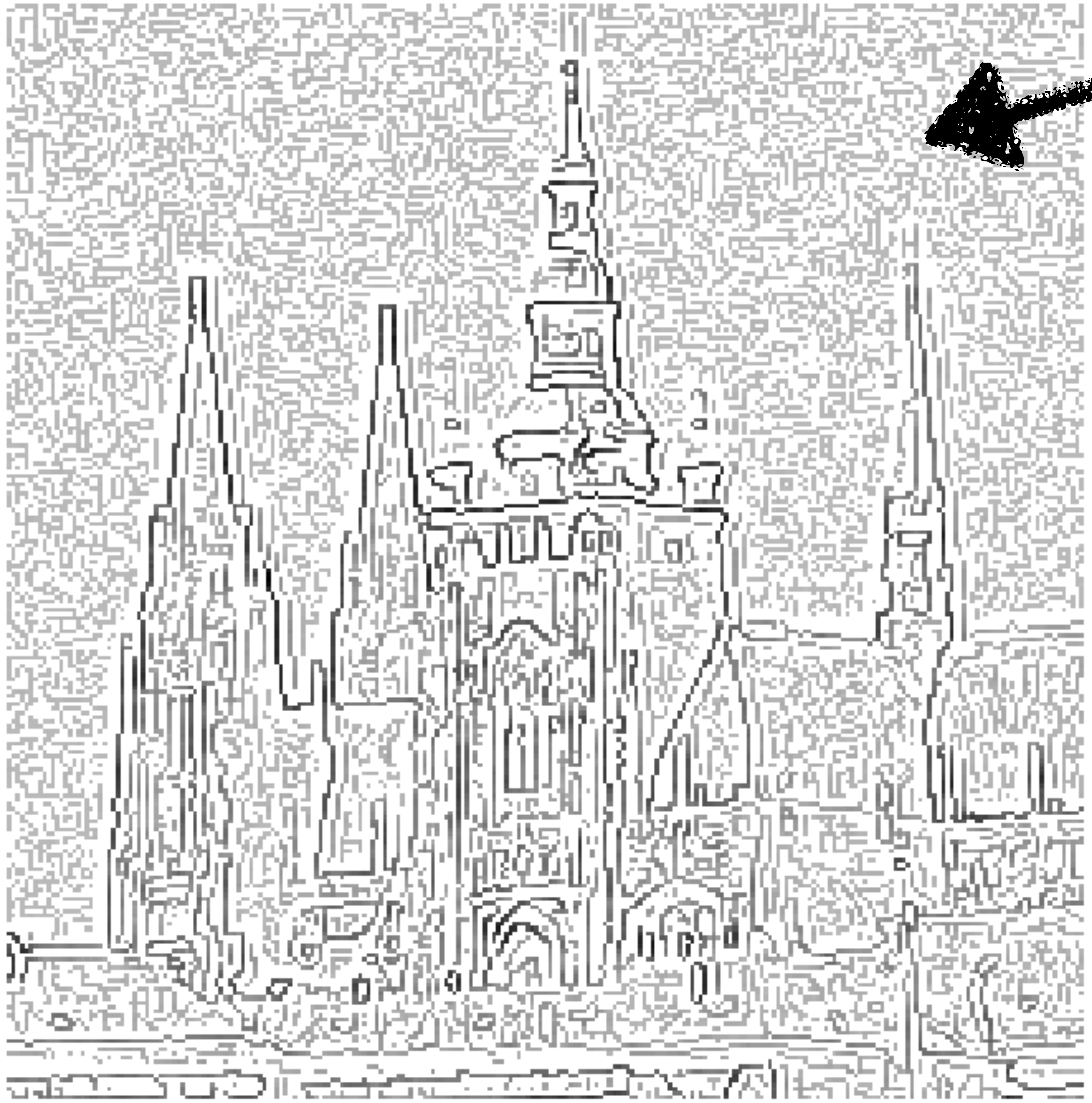
Non-Maximum Suppression & Hysteresis



Non-maximal suppression

slide credit: Václav Hlaváč

Non-Maximum Suppression & Hysteresis

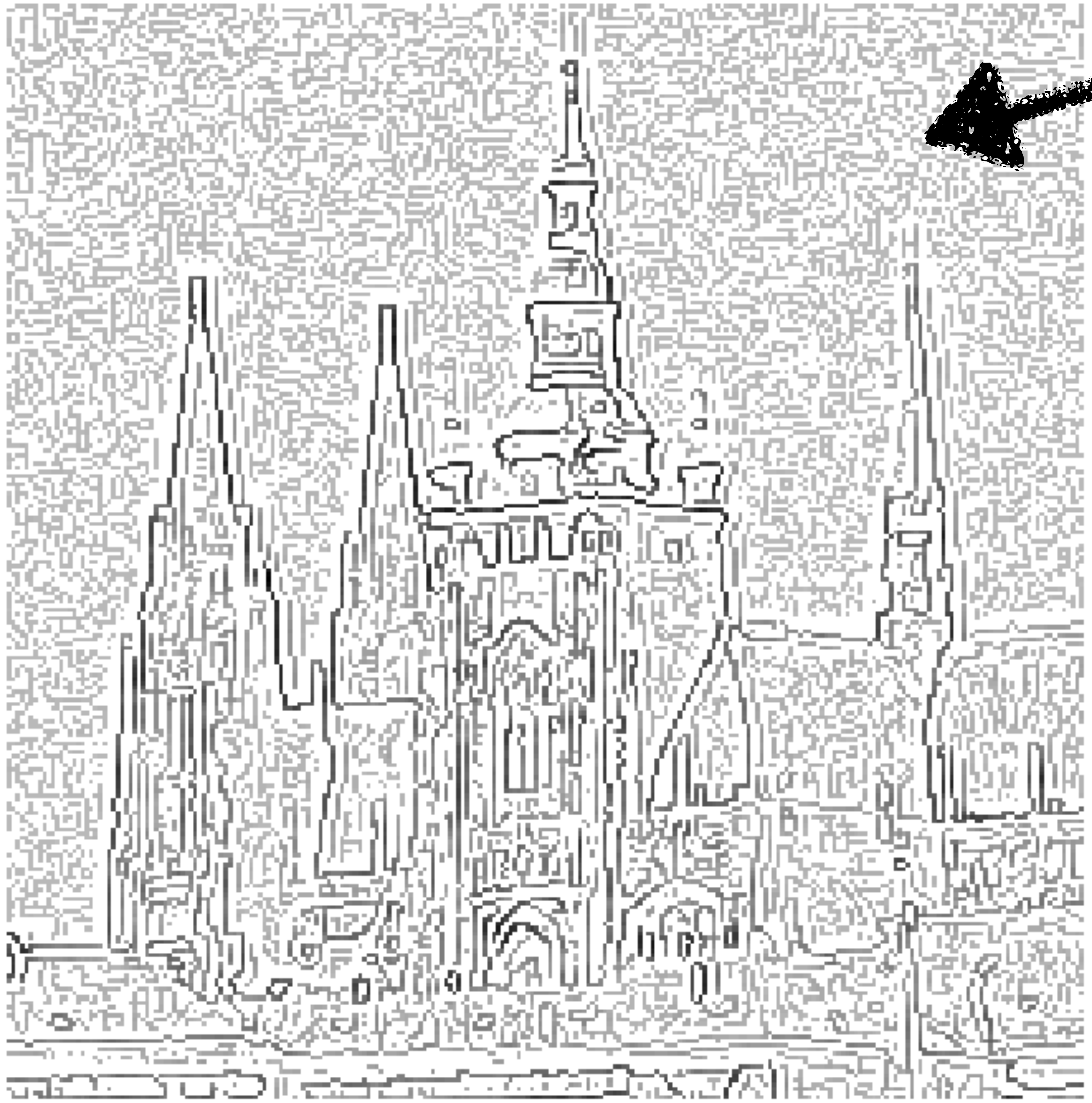


Non-maximal suppression

lots of noisy edges

slide credit: Václav Hlaváč

Non-Maximum Suppression & Hysteresis



Non-maximal suppression

lots of noisy edges

Hysteresis:

slide credit: Václav Hlaváč

Non-Maximum Suppression & Hysteresis

Non-maximal suppression

lots of noisy edges

Hysteresis:

- Start with edge pixel with edge score above higher threshold t_{high}

slide credit: Václav Hlaváč

Non-Maximum Suppression & Hysteresis

Non-maximal suppression

lots of noisy edges

Hysteresis:

- Start with edge pixel with edge score above higher threshold t_{high}
- Follow edge as long as pixels have edge score above lower threshold t_{low}

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Non-Maximum Suppression & Hysteresis

Non-maximal suppression

Hysteresis:

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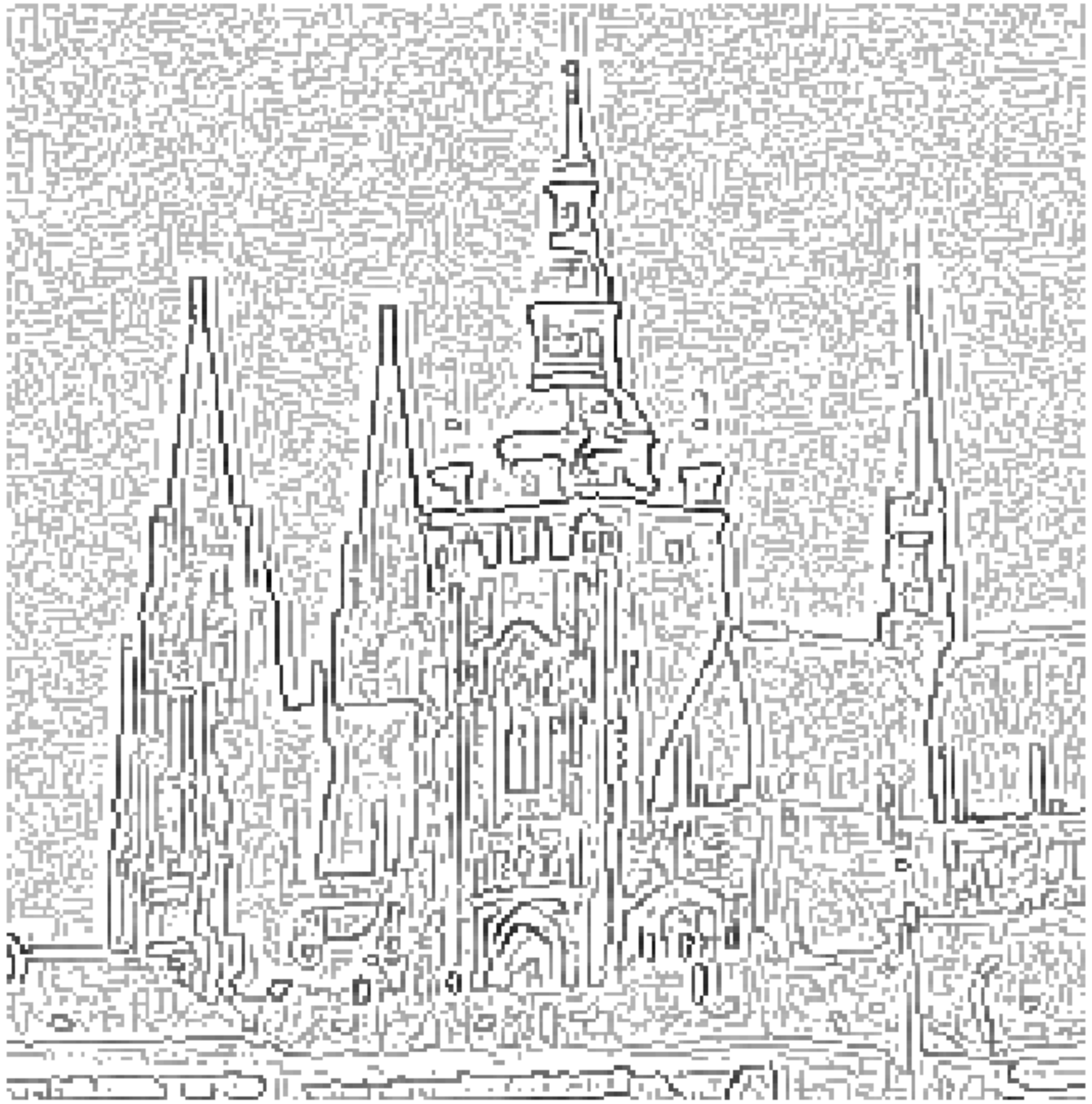
- Iterate until all pixels considered

slide credit: Václav Hlaváč

Edge-Based Image Segmentation



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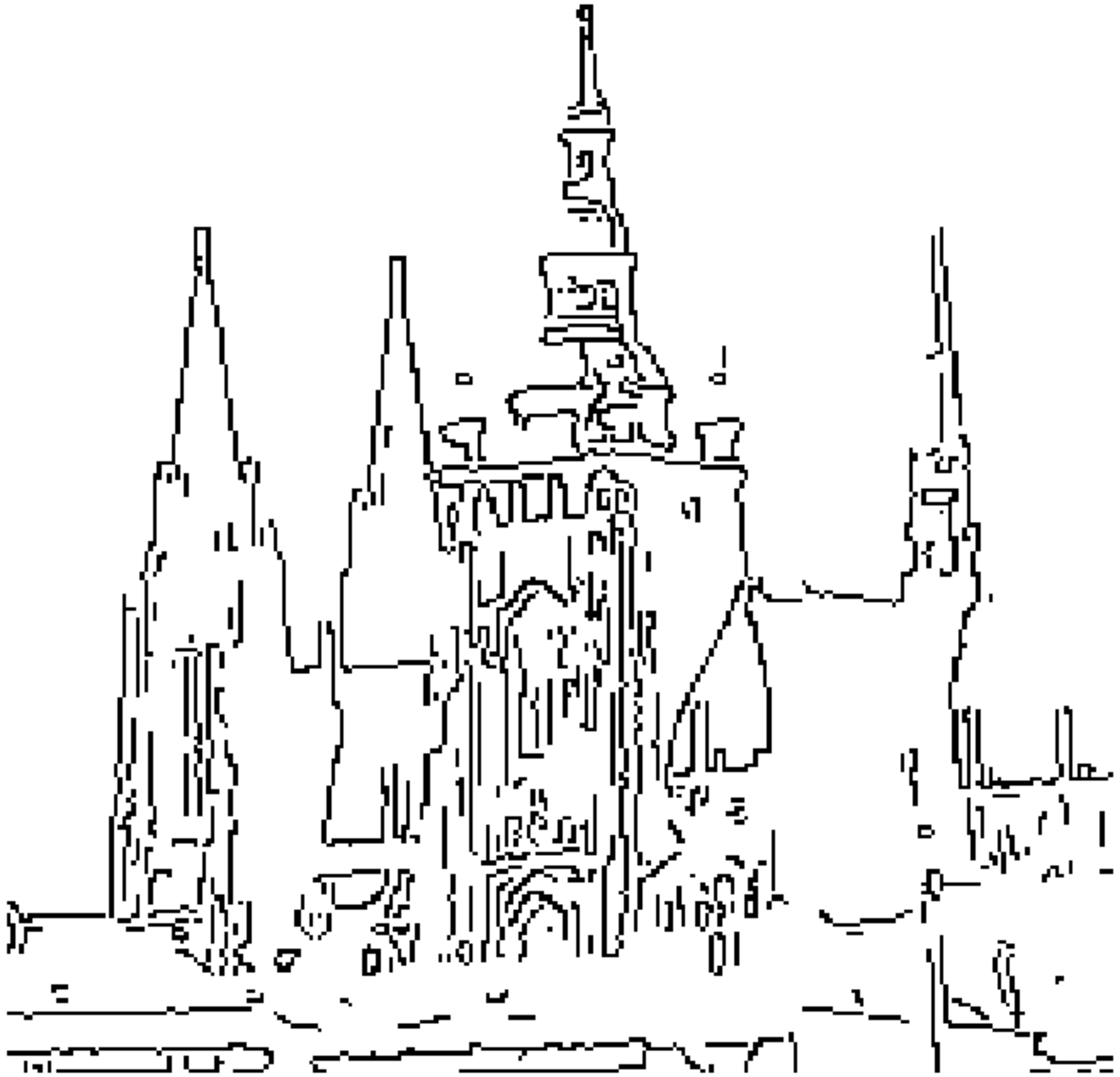
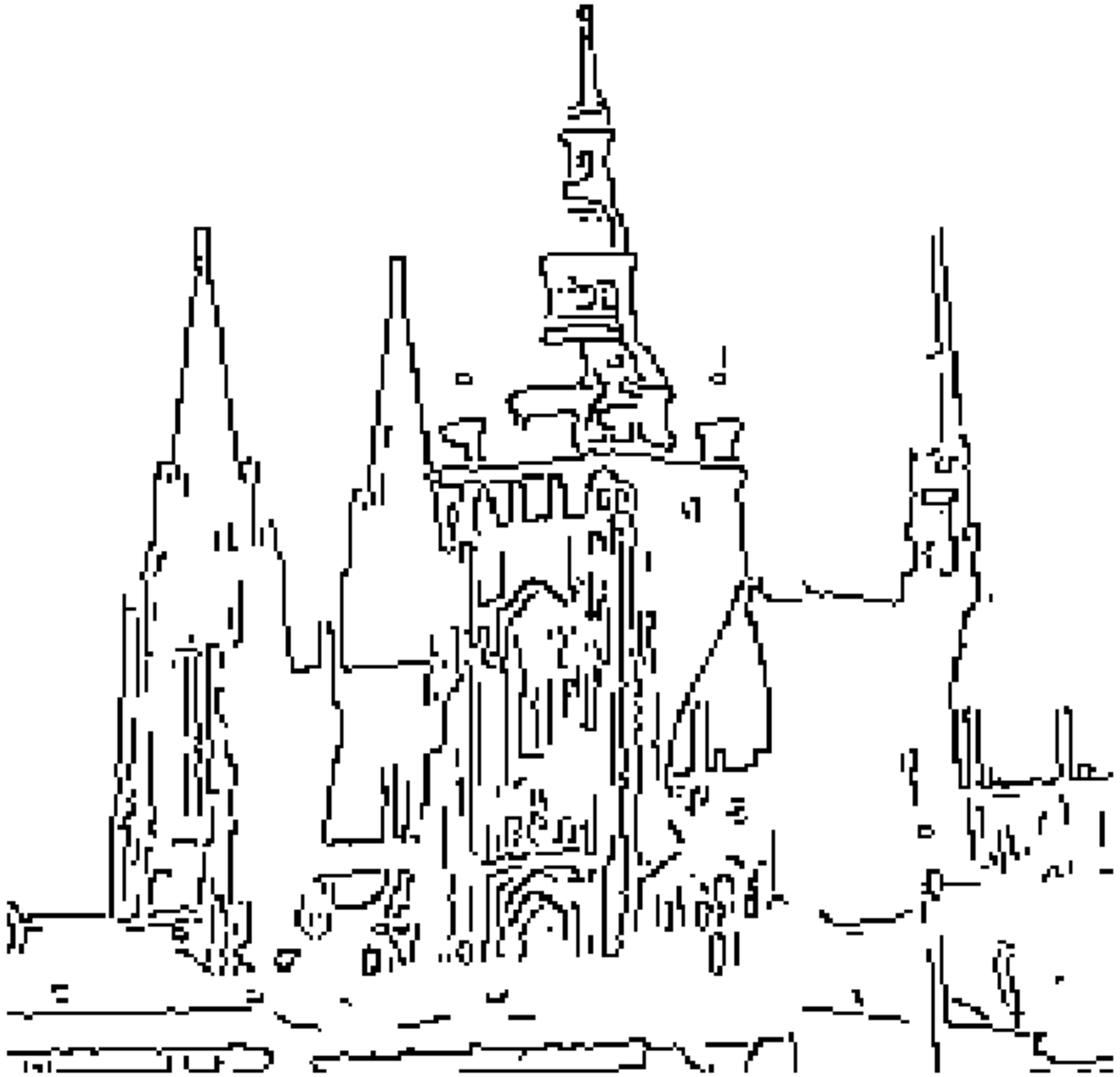


hysteresis
high threshold 70, lower 10

slide credit: Václav Hlaváč

Edge Relaxation



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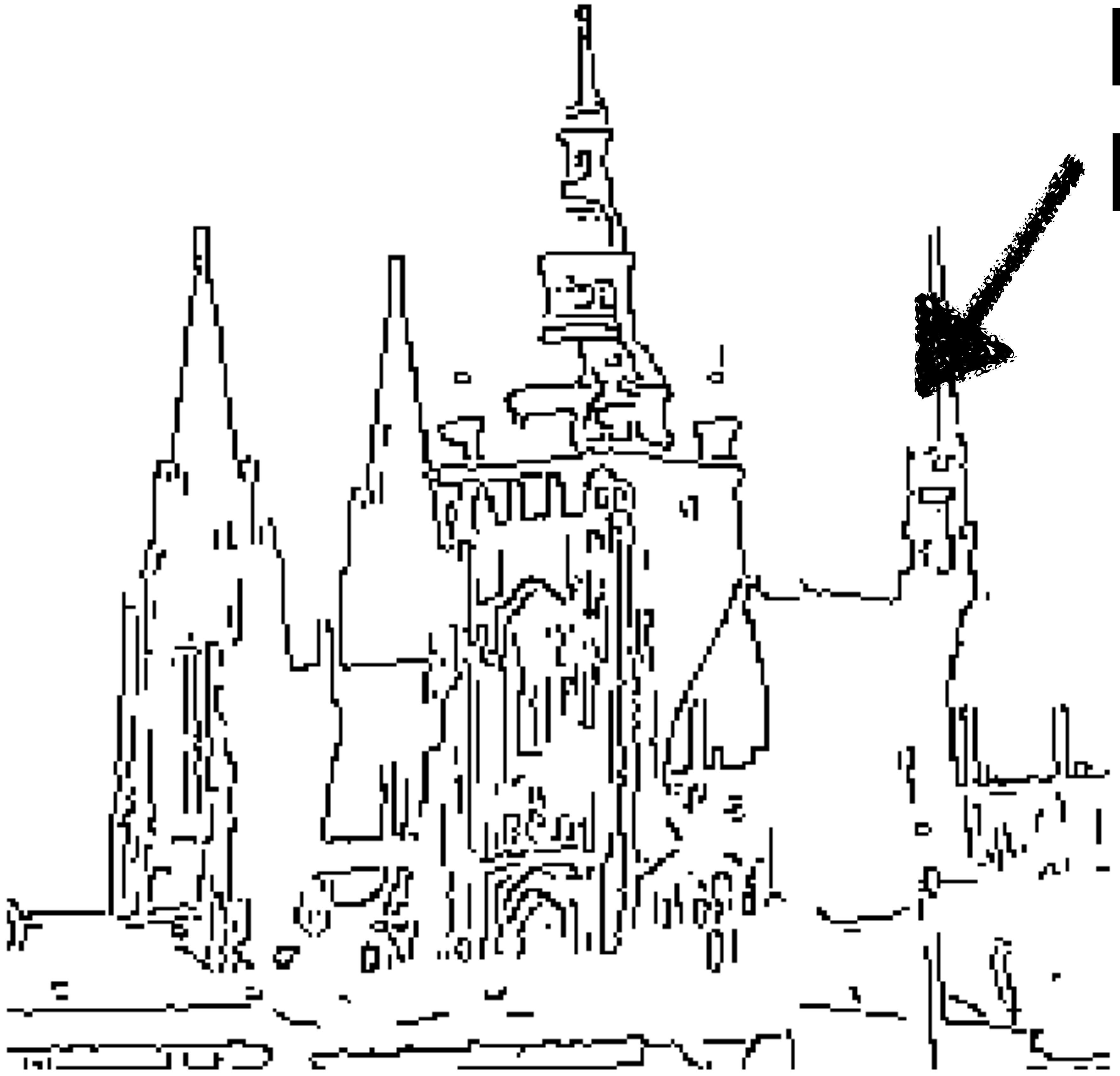
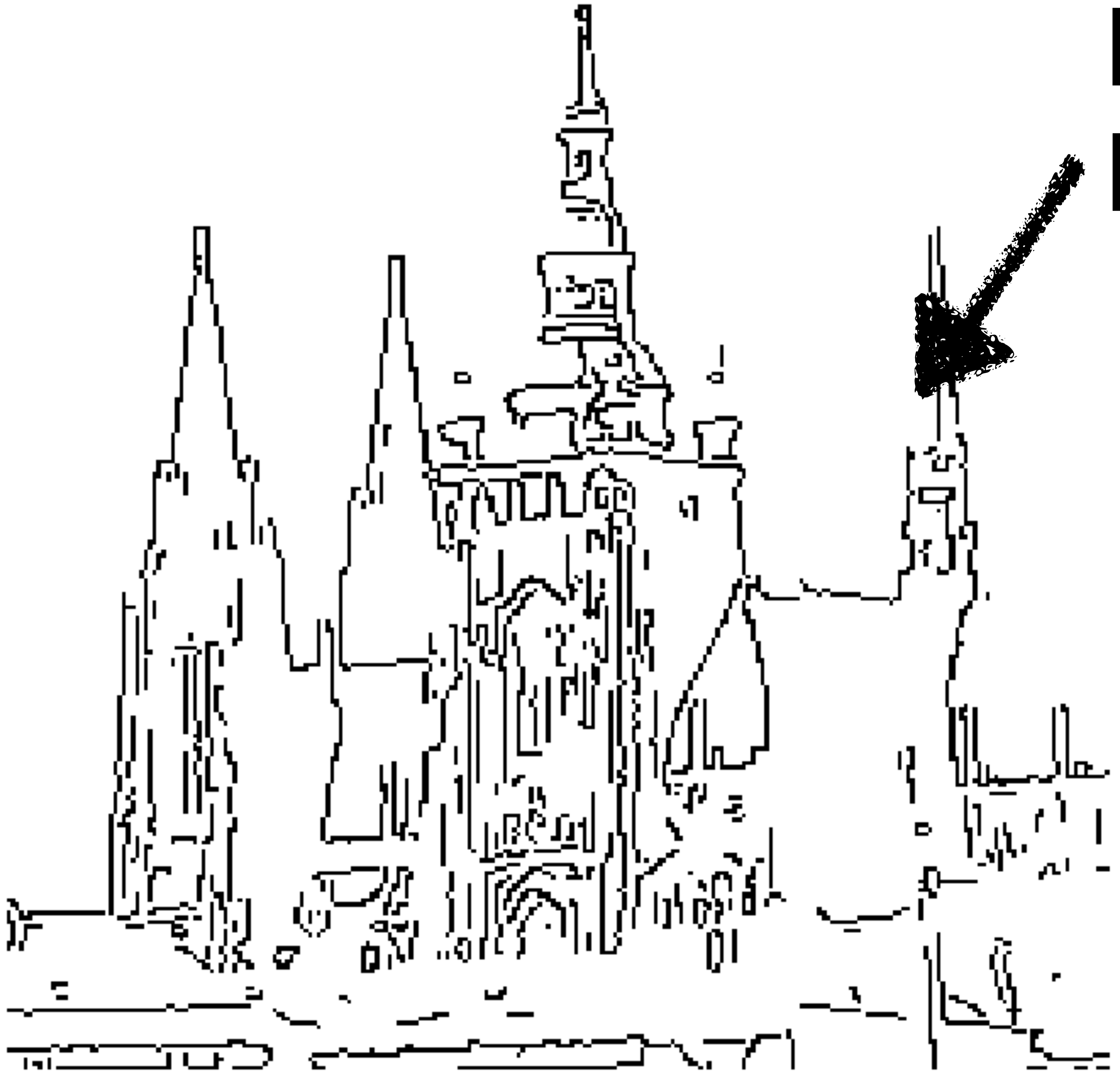


hysteresis
high threshold 70, lower 10

No closed boundaries
Parts missing

slide credit: Václav Hlaváč

Edge Relaxation



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slide credit: Václav Hlaváč

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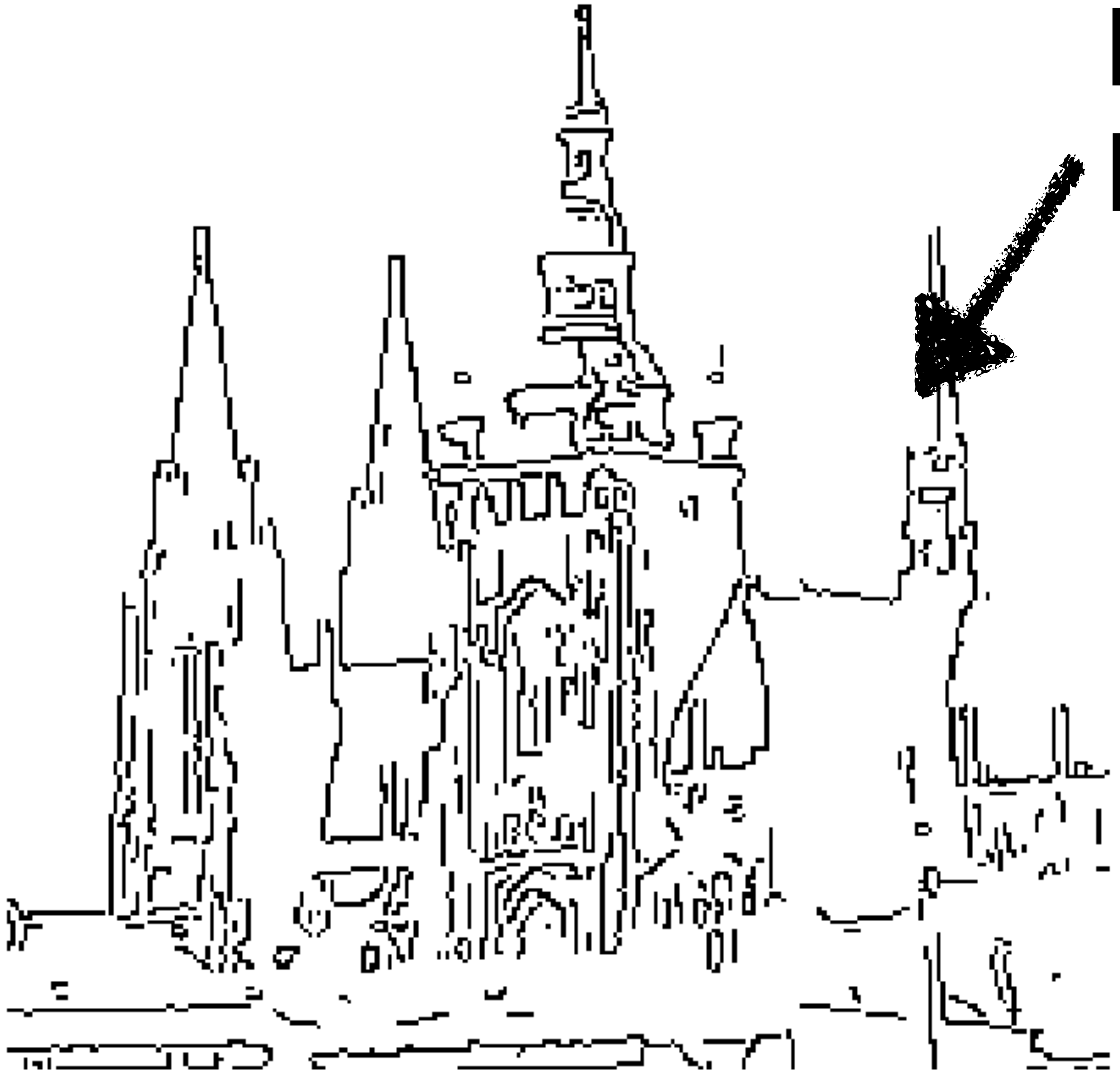

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Edge Relaxation:

- Attempt to close gaps in post-processing

slide credit: Václav Hlaváč

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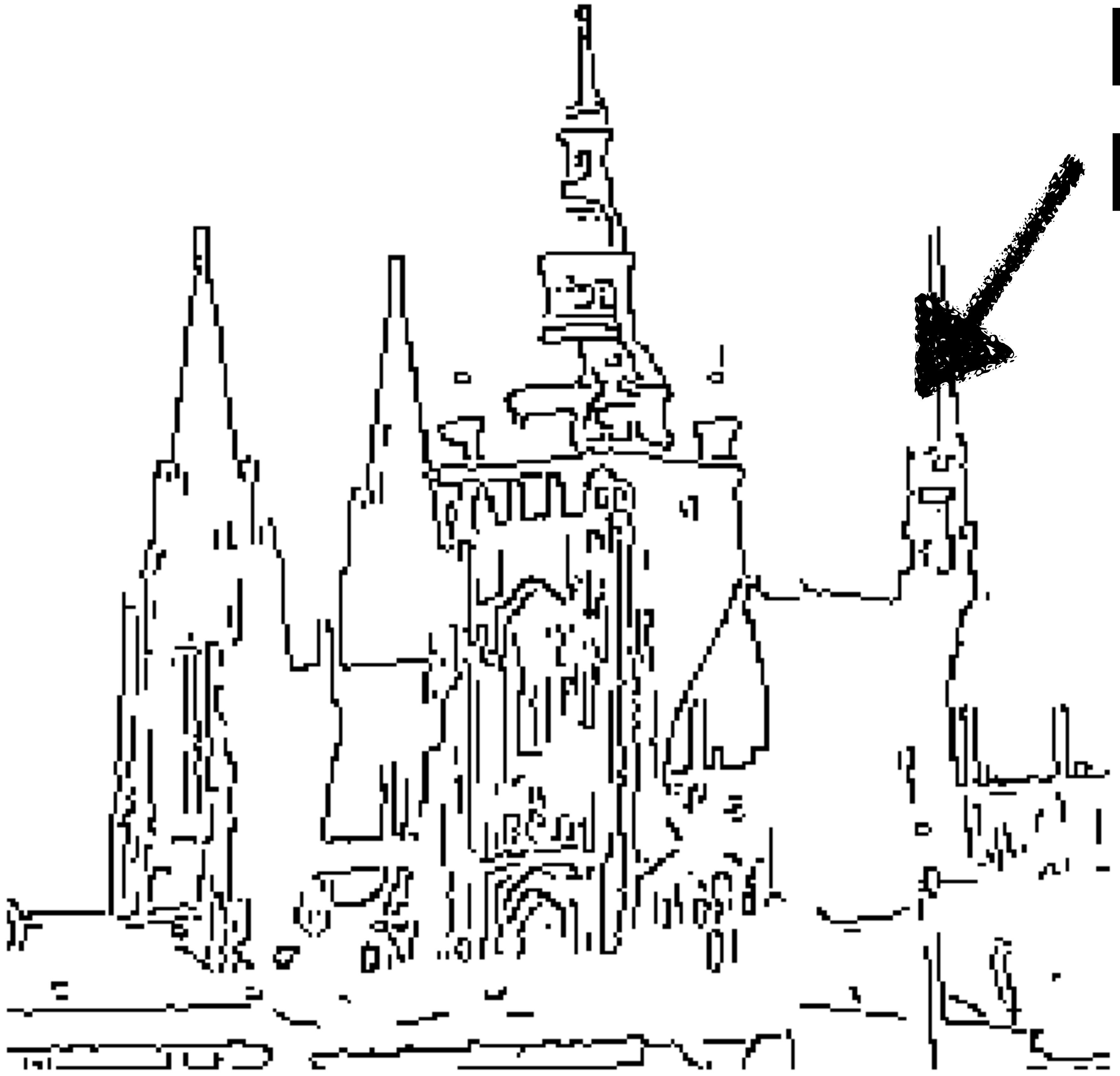

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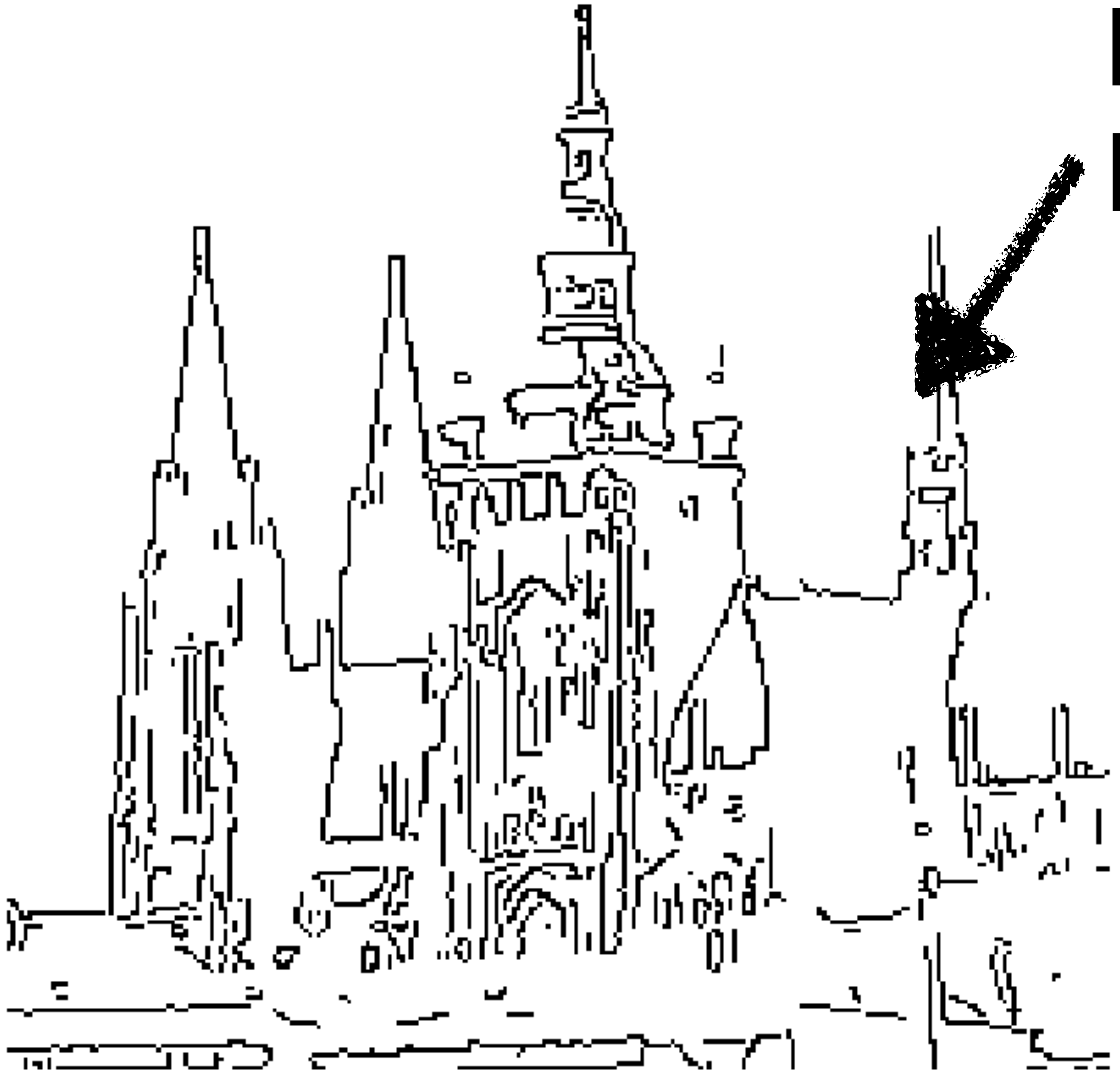
Edge Relaxation:

- Attempt to close gaps in post-processing
- Iteratively improve edge properties based on neighboring edges

slide credit: Václav Hlaváč

Edge Relaxation



No closed boundaries
Parts missing

hysteresis
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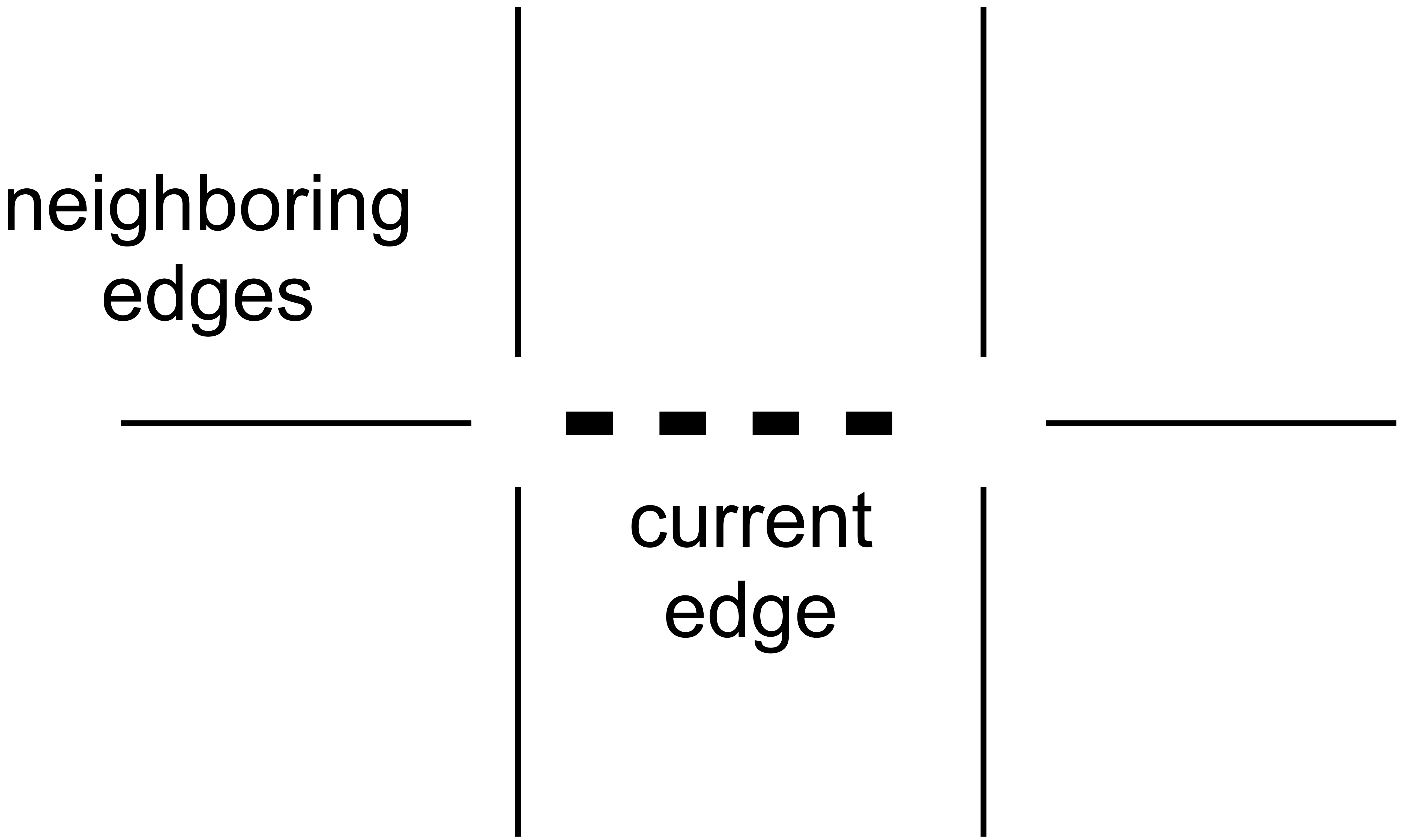
Edge Relaxation:

- Attempt to close gaps in post-processing
- Iteratively improve edge properties based on neighboring edges
- Instance of a general algorithm "relaxation labelling"

slide credit: Václav Hlaváč

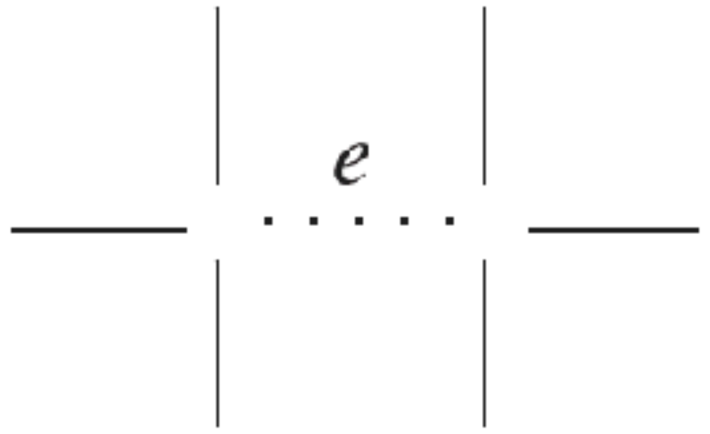
Edge Relaxation For Crack Edges

Example illustration here based on crack edges (Hanson, Rieseman, 1978)



slide credit: Václav Hlaváč

Crack Edges Patterns

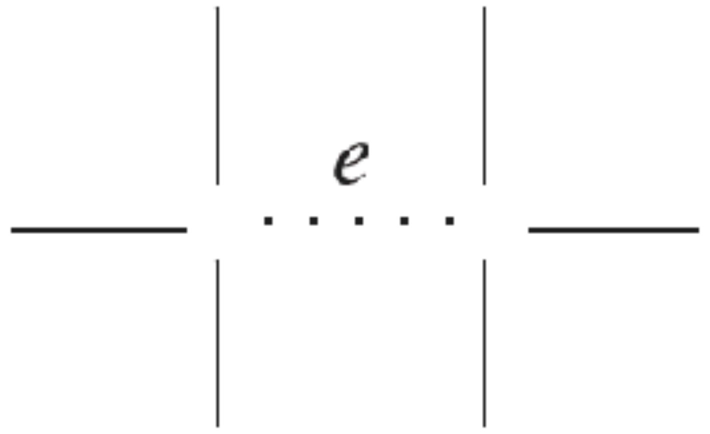


0–0

- **0-0** isolated edge → decrease edge confidence

slide credit: Václav Hlaváč

Crack Edges Patterns

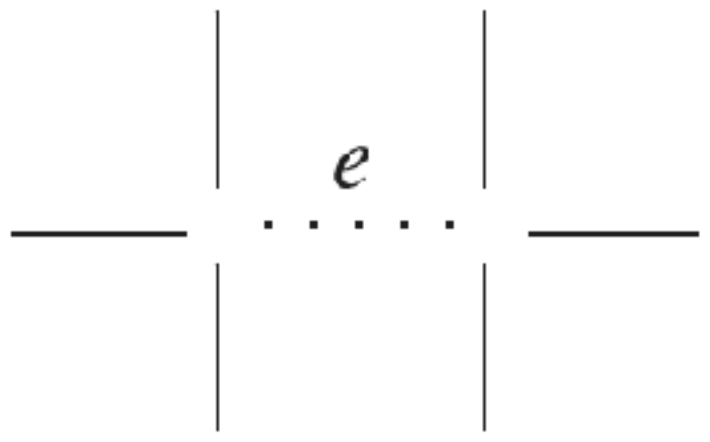


0–0

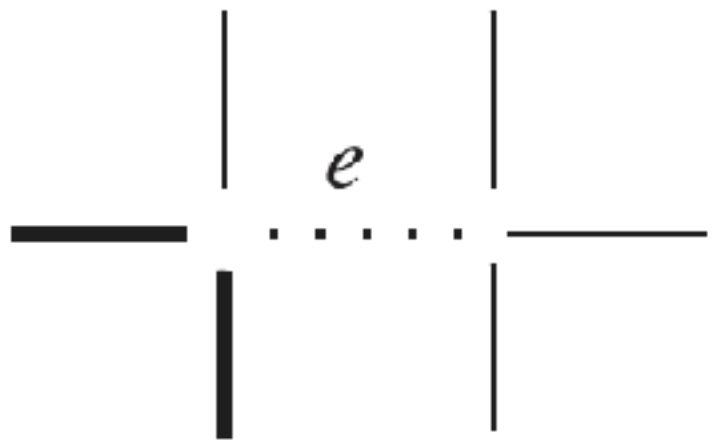
- **0-0** isolated edge → decrease edge confidence
- **0-1** uncertain → weak increase, or no influence

slide credit: Václav Hlaváč

Crack Edges Patterns



0–0

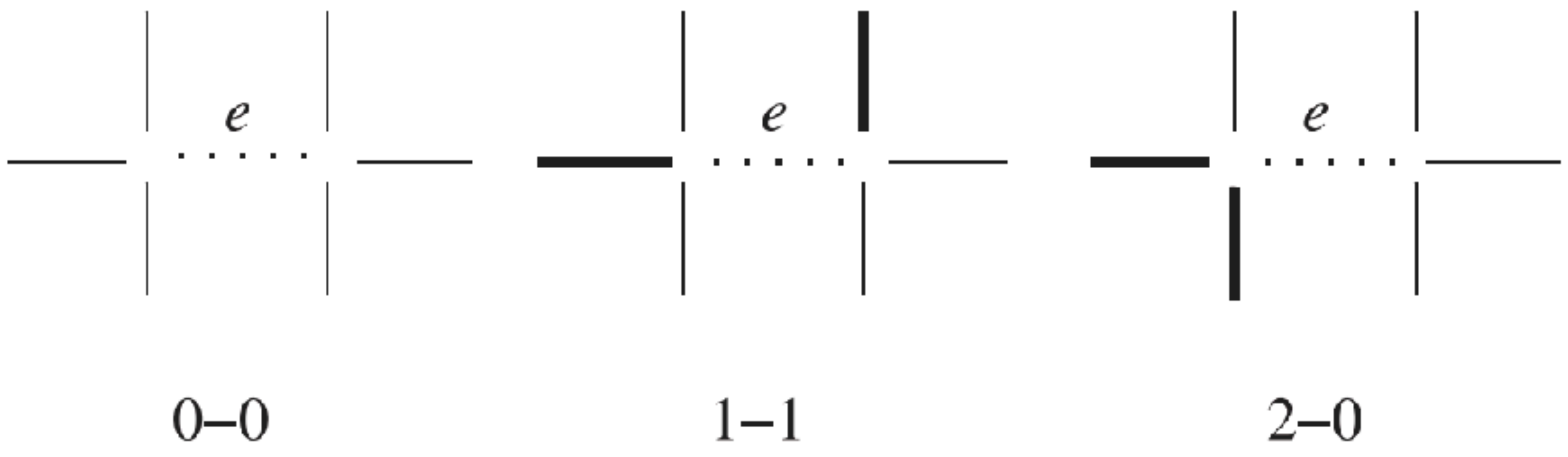


2–0

- **0-0** isolated edge → decrease edge confidence
- **0-1** uncertain → weak increase, or no influence
- **0-2**, **0-3** dead end → decrease edge confidence

slide credit: Václav Hlaváč

Crack Edges Patterns



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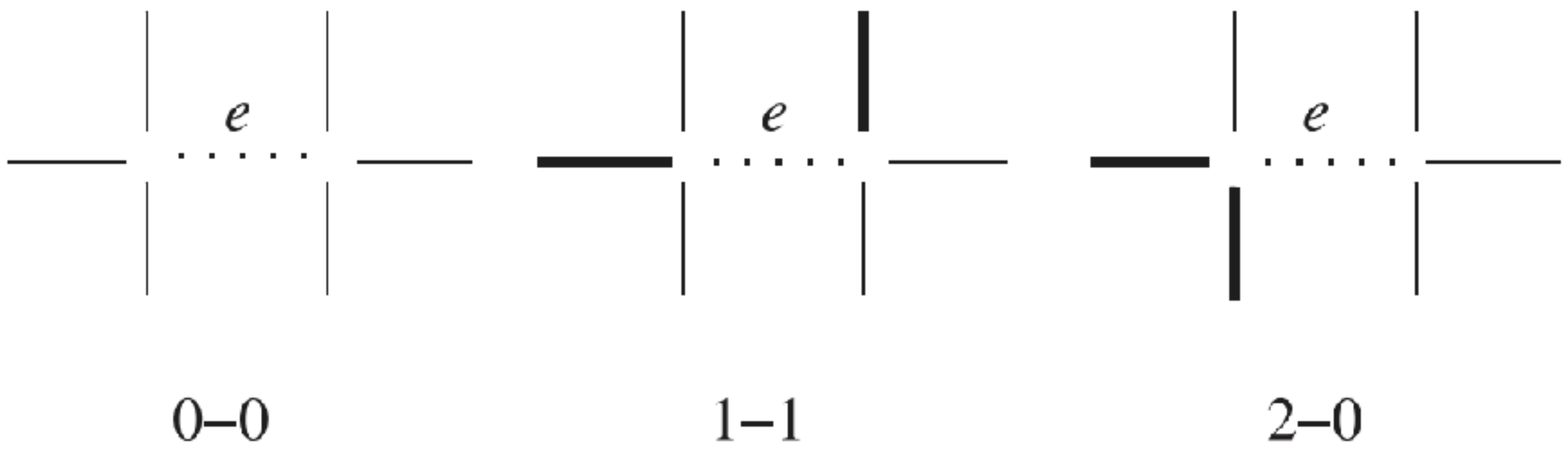

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slide credit: Václav Hlaváč

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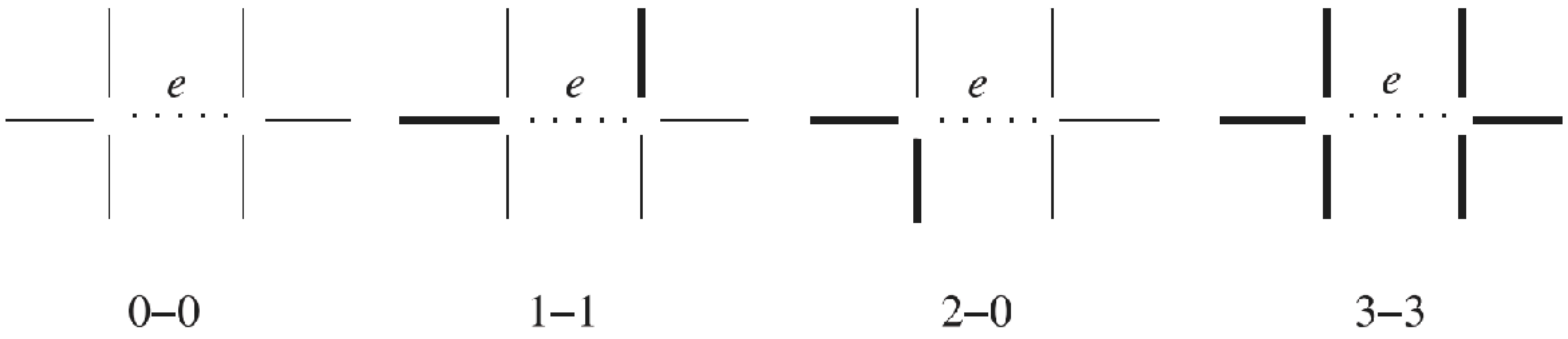

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- **0-2**, **0-3** dead end → decrease edge confidence
- **1-1** continuation → strong positive influence on edge confidence
- **1-2**, **1-3** continuation to border intersection → medium positive influence on edge confidence
- **2-2**, **2-3**, **3-3** bridge between borders → not necessary for segmentation, no influence on edge confidence

slide credit: Václav Hlaváč

Edge Relaxation

Evaluate confidence $c^1(e)$ for each crack edge e

Set k = 1

Repeat

Determine type of edge based on confidences $c^k(e)$ in neighborhood

Compute confidence $c^{k+1}(e)$ of e based on type and $c^k(e)$

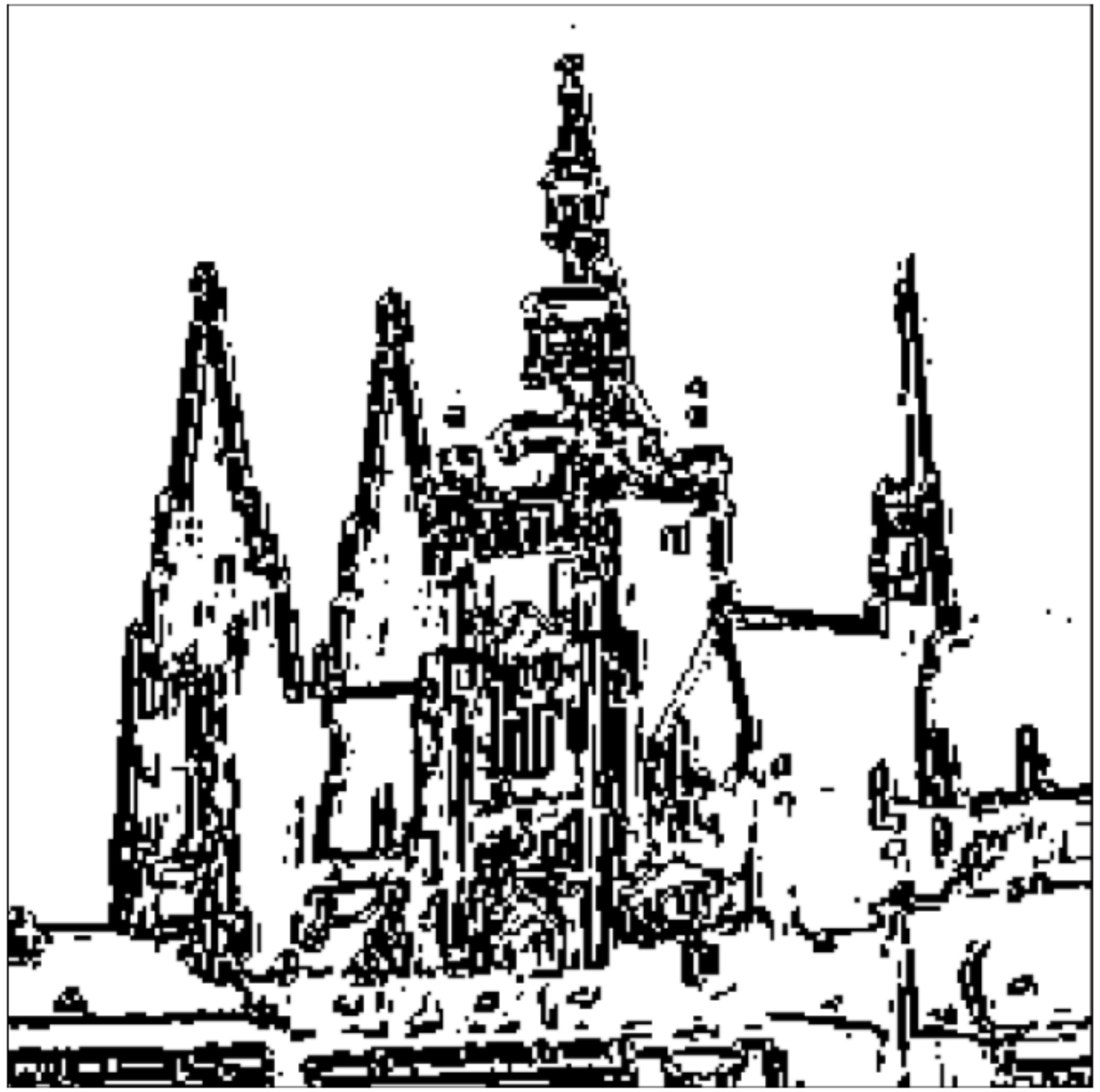
Evaluate model on all 2D-2D and 2D-3D matches

k = k + 1

Stop if all edge confidences have converged to 0 or 1

slide credit: Václav Hlaváč

Edge Relaxation



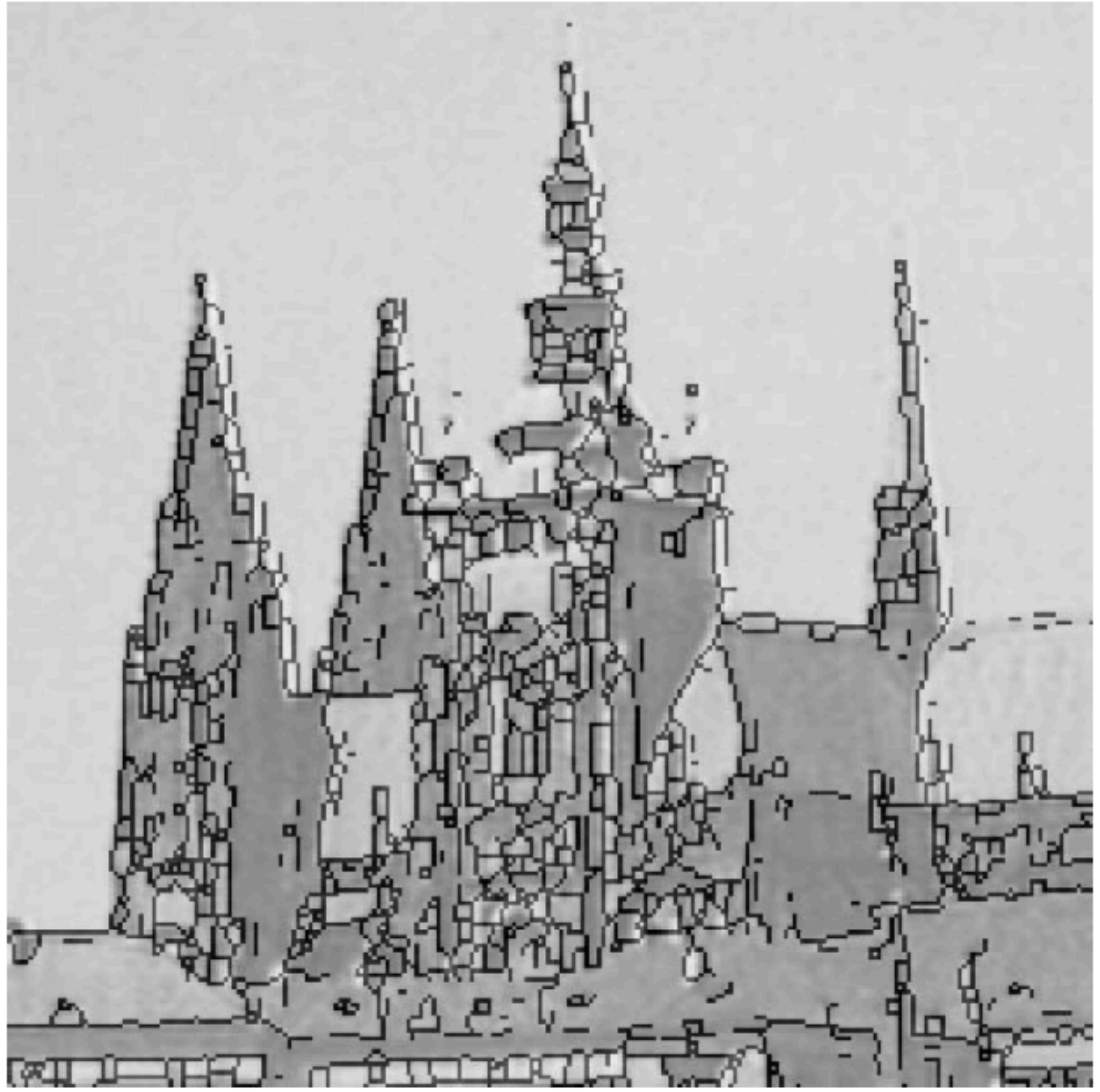
borders after 10 iterations

borders after thinning

slide credit: Václav Hlaváč

Edge Relaxation

borders after 100 iterations
thinned



overlaid over original

slide credit: Václav Hlaváč

Region Growing

Repeat until no more seeds

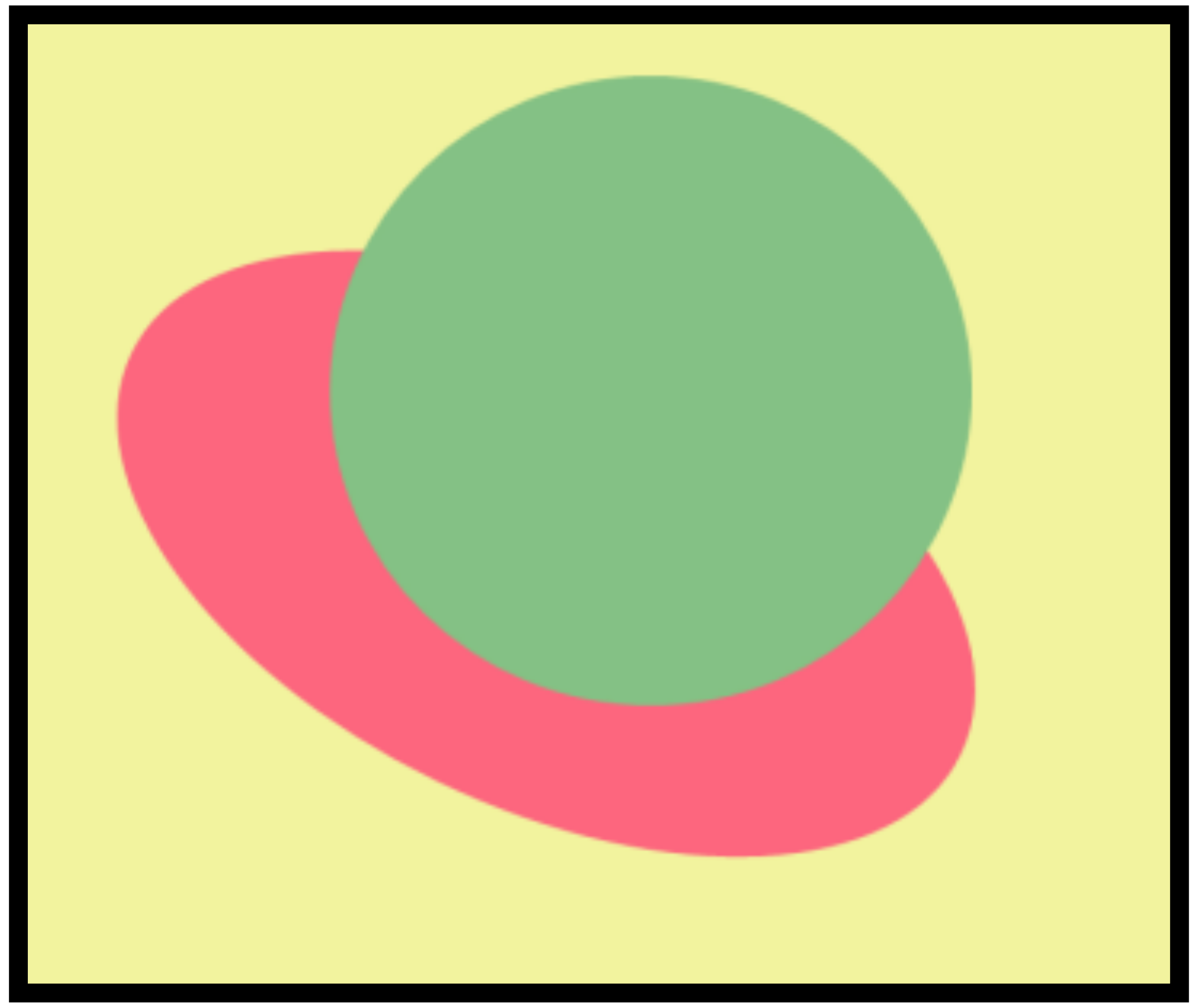
Select set K of seed pixels to start new region R

Repeat

For each pixel $p \in K$:

Add neighbor q to K and R if similar enough to pixels in R

remove p from K



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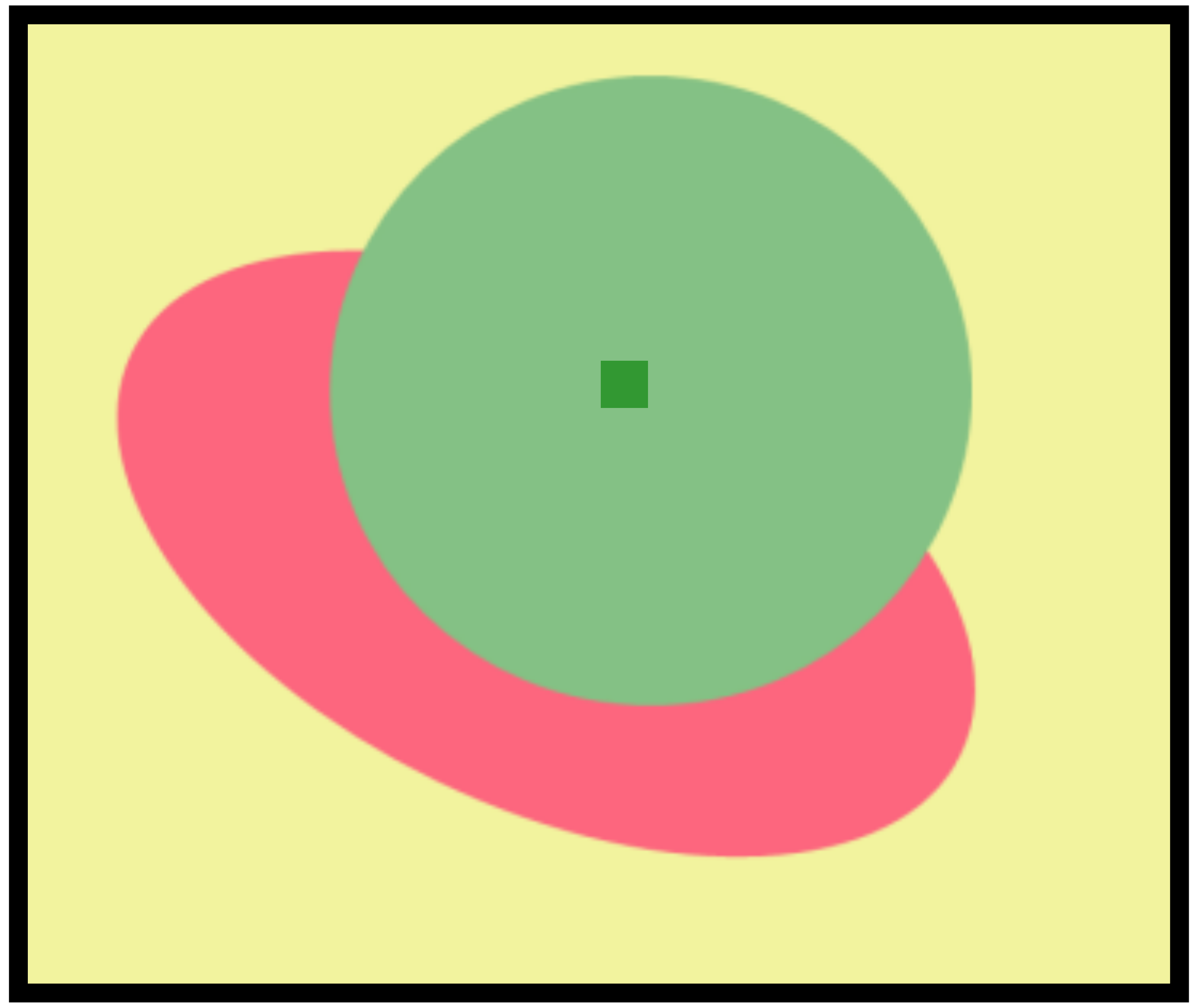

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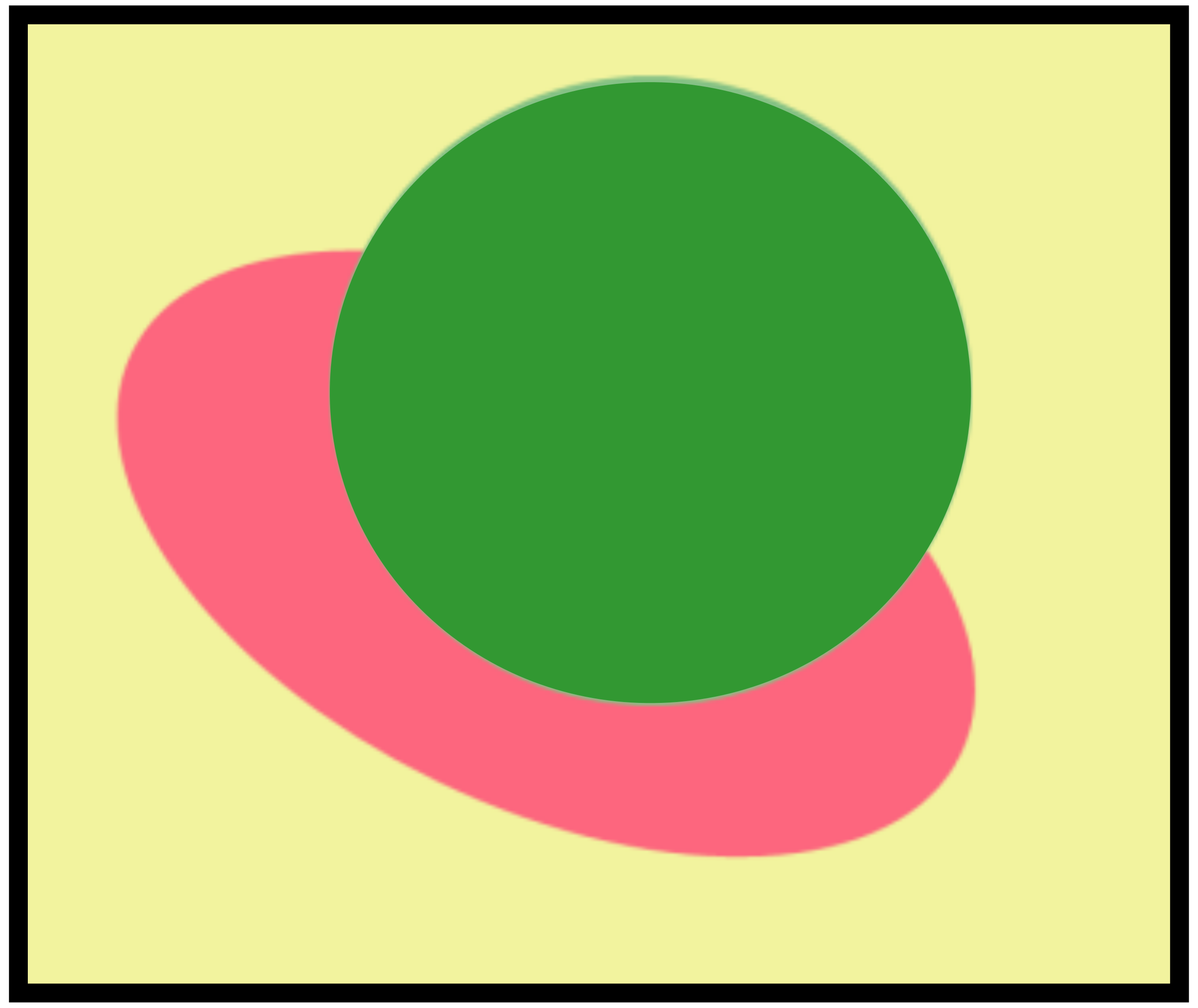
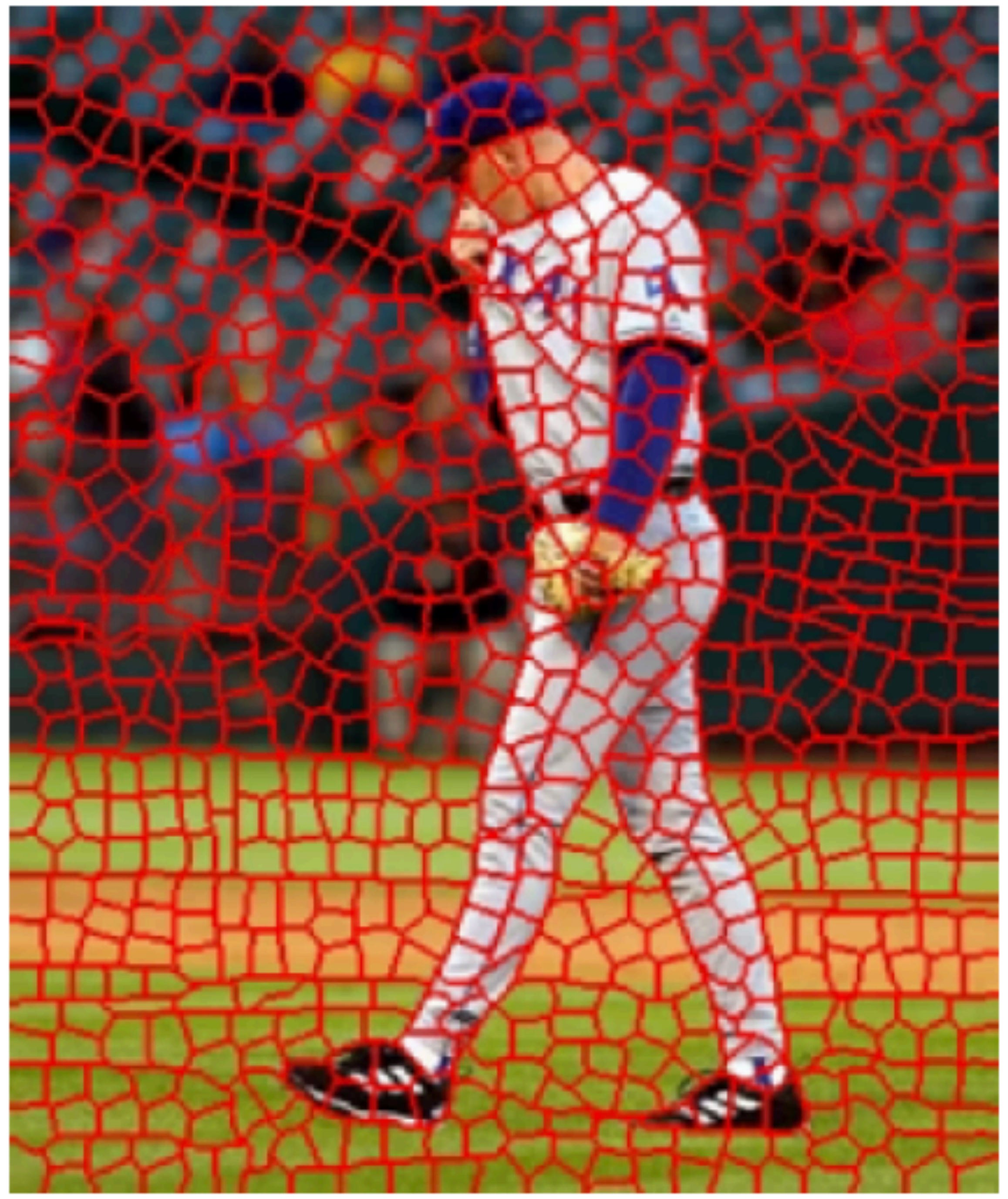


slide credit: Václav Hlaváč

Superpixels

- Group together similar-looking pixels
- Increases efficiency of further processing: use superpixels rather than pixels



[Ren & Malik, Learning a classification model for segmentation, ICCV 2003]

slide credit: Václav Hlaváč

Discussion

- Edge-based segmentation and region growing are examples for **bottom-up approaches** to segmentation:
  - Obtain initial segmentation, then process it (merge close-by segments to semantically meaningful regions, etc.)
  - Risk: early mistakes cannot be recovered (premature hard decisions)

slide credit: Václav Hlaváč

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- Edge-based segmentation and region growing are examples for **bottom-up approaches** to segmentation:
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slide credit: Václav Hlaváč

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  - Risk: early mistakes cannot be recovered (premature hard decisions)
- **Top-down segmentation**: start with larger regions and split them into semantically meaningful parts
- Bottom-up and top-down approaches can work together, are not mutually exclusive

slide credit: Václav Hlaváč

Lecture Overview

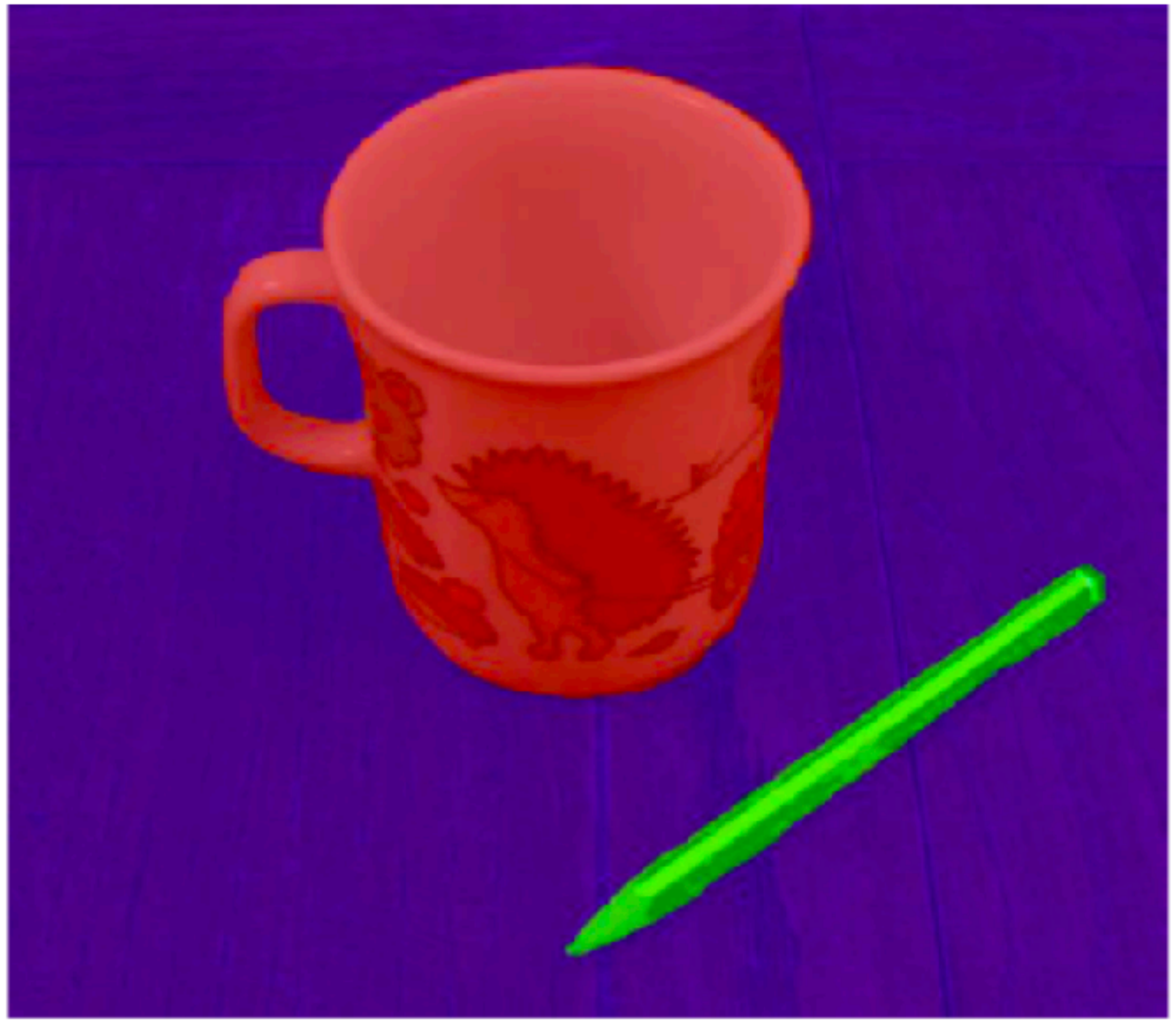
simple & heuristic

- A simple approach to segmentation: **(intensity) thresholding**
- Segmentation based on spatial coherence: **edge-based segmentation**, **region growing**
- Segmentation as a **clustering** problem: **k-means clustering**, **mean-shift clustering**
- Segmentation as a statistical (unsupervised) learning problem: **expectation maximization (EM) algorithm**
- Next lecture: **graph-based segmentation**, **supervised learning with neural networks** (if time and interest)

complex & principled

slide credit: Václav Hlaváč

Segmentation As Clustering



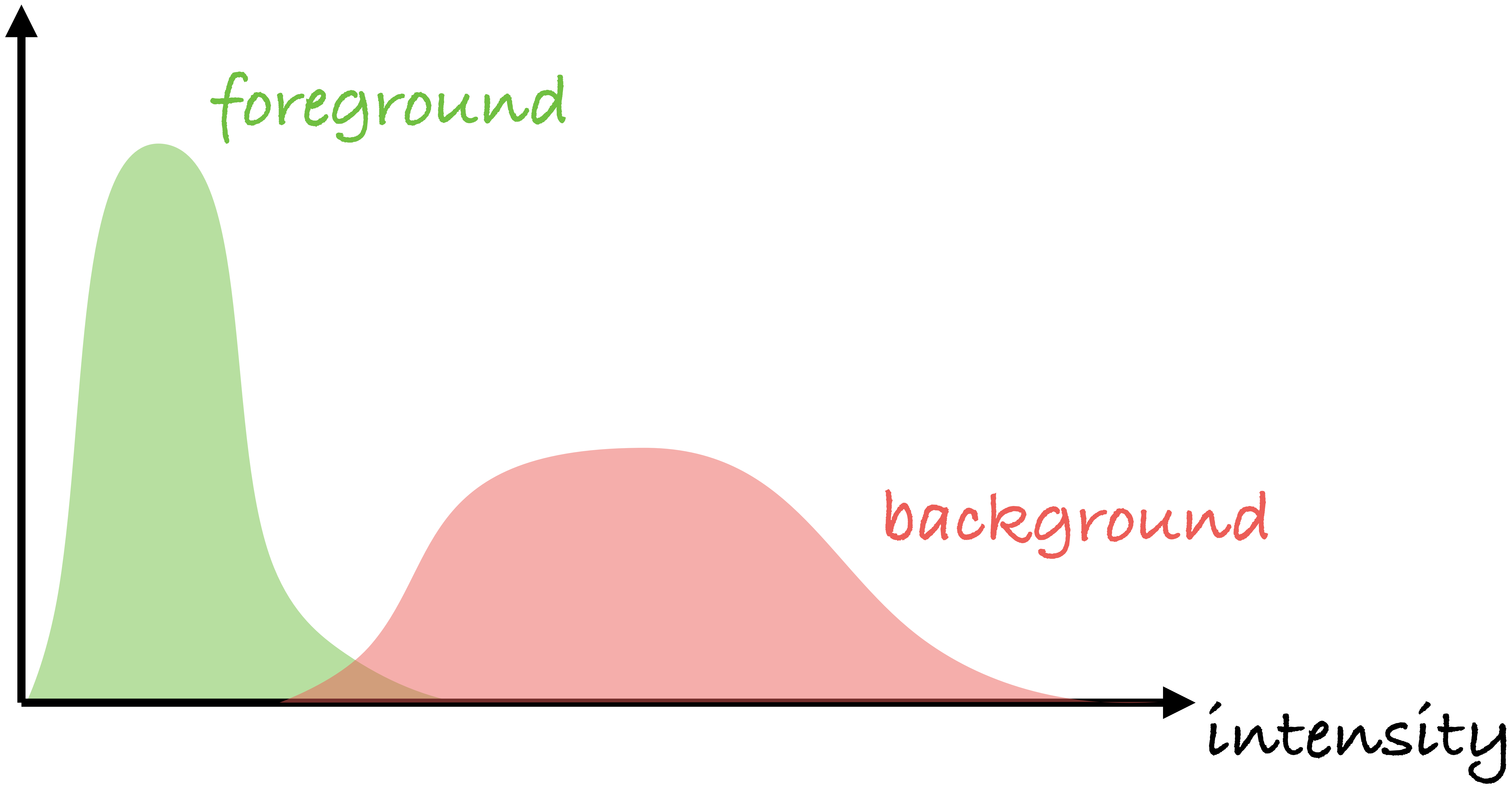
Image, courtesy Ondřej Drbohlav

Goal: **cluster pixels** into **regions** based on spatial and appearance similarity

slide credit: Václav Hlaváč

Thresholding As Clustering

Goal: find **clusters of pixels that belong together** based on pixel intensities



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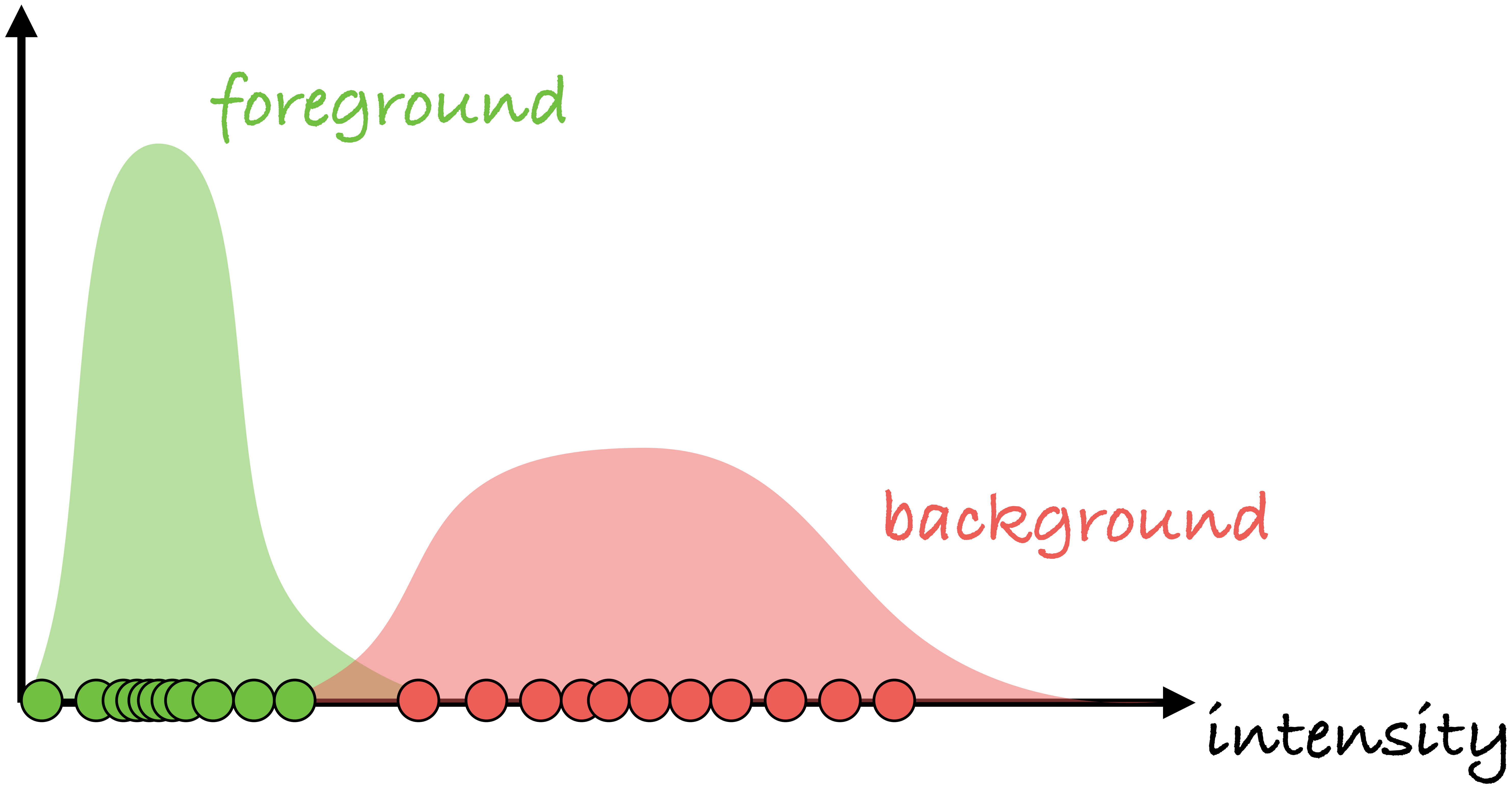

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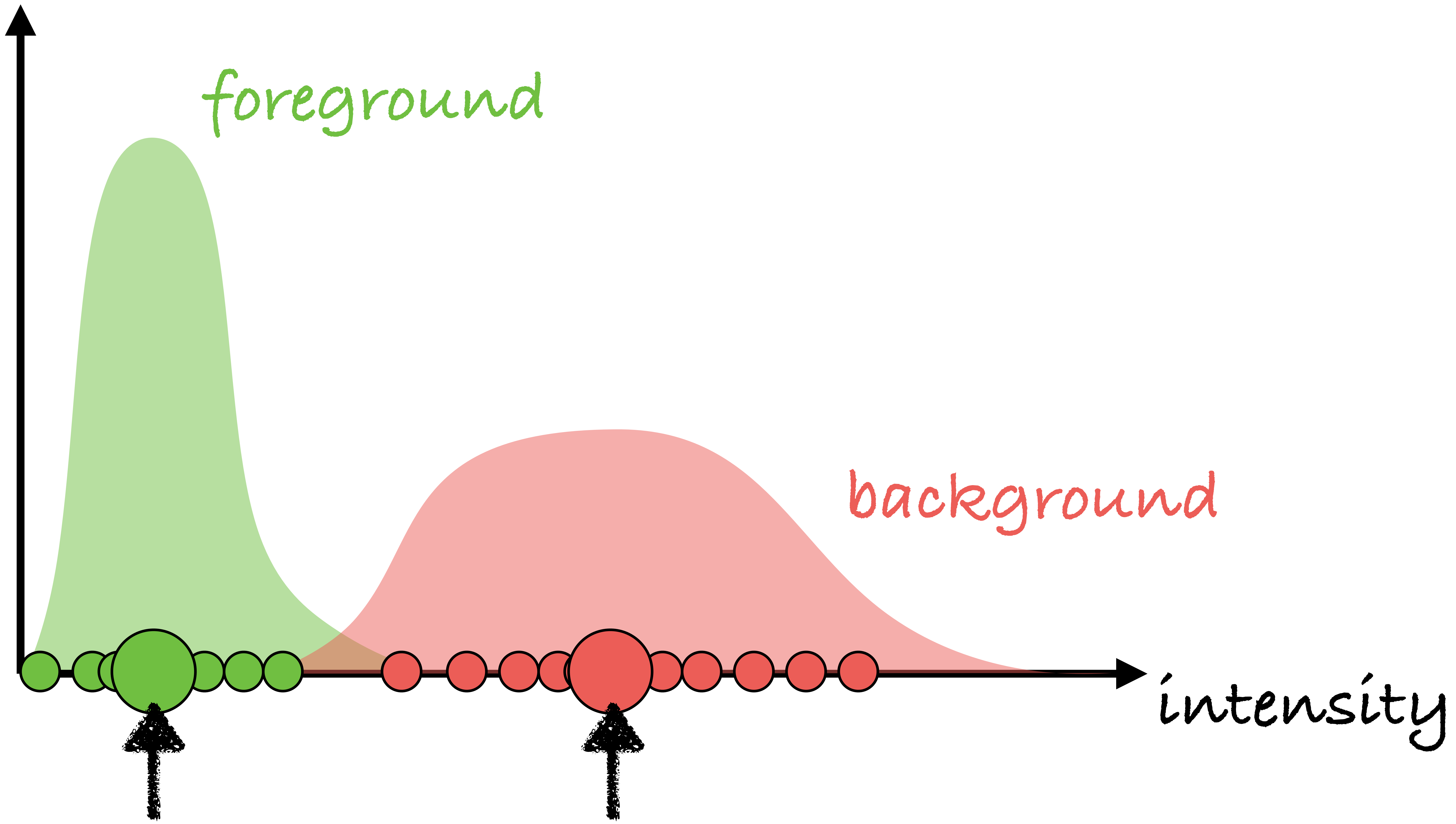

Represent clusters by “centers” assign pixel to cluster of nearest center

slide credit: Bastian Leibe

k-Means Clustering

- **Chicken-and-egg problem**:
  - Given cluster membership, we can compute cluster centers (e.g., as mean)

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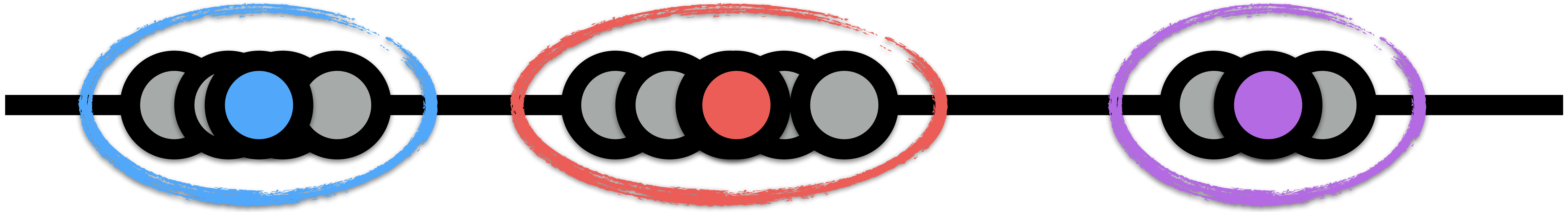

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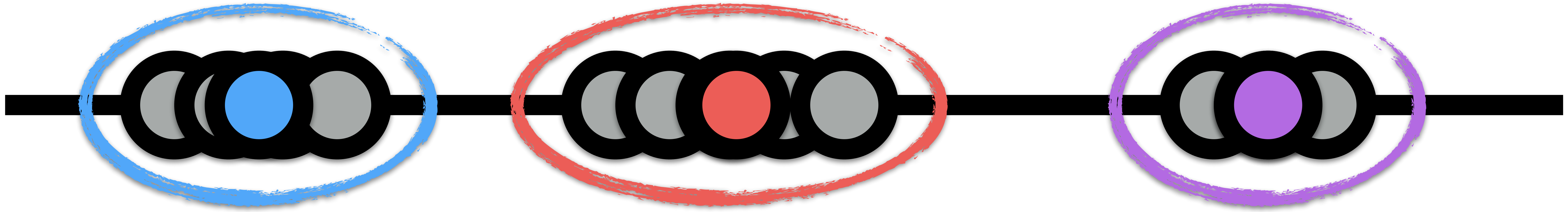

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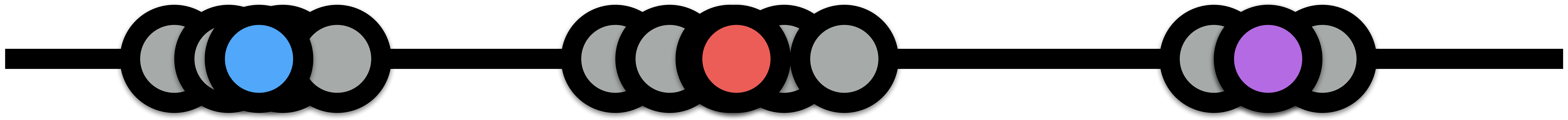

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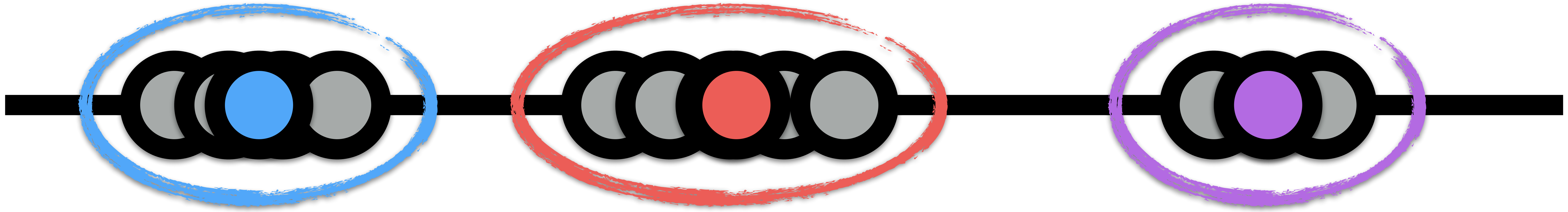

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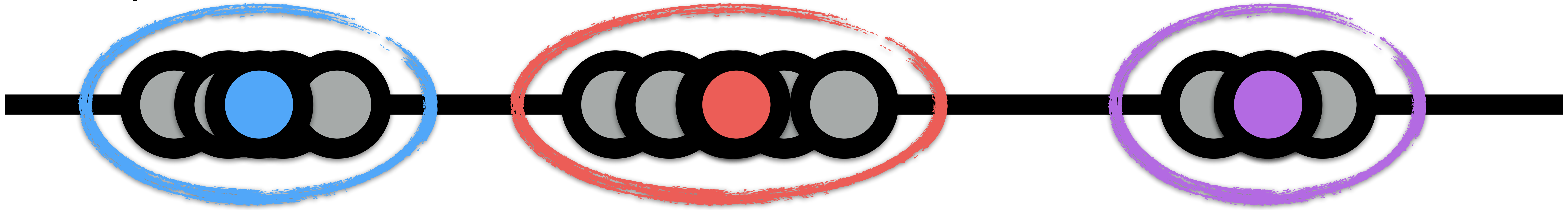

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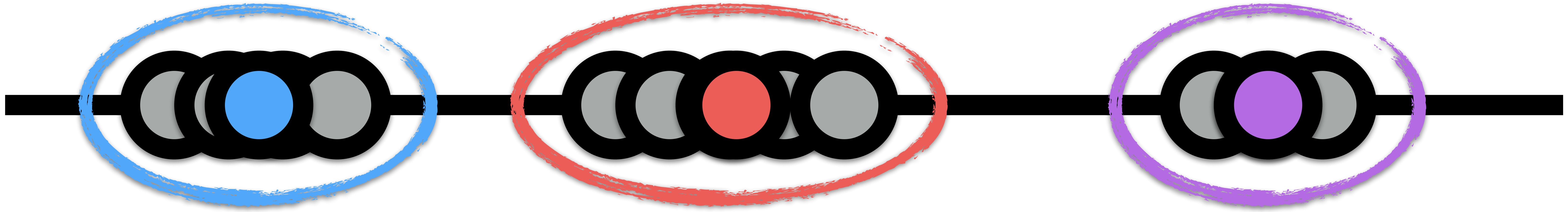

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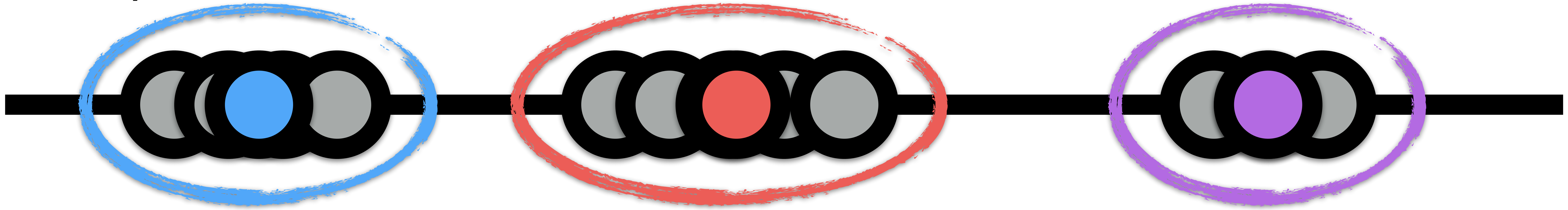


- **k-means clustering**: alternate between computing cluster membership and computing cluster centers

slide credit: Bastian Leibe

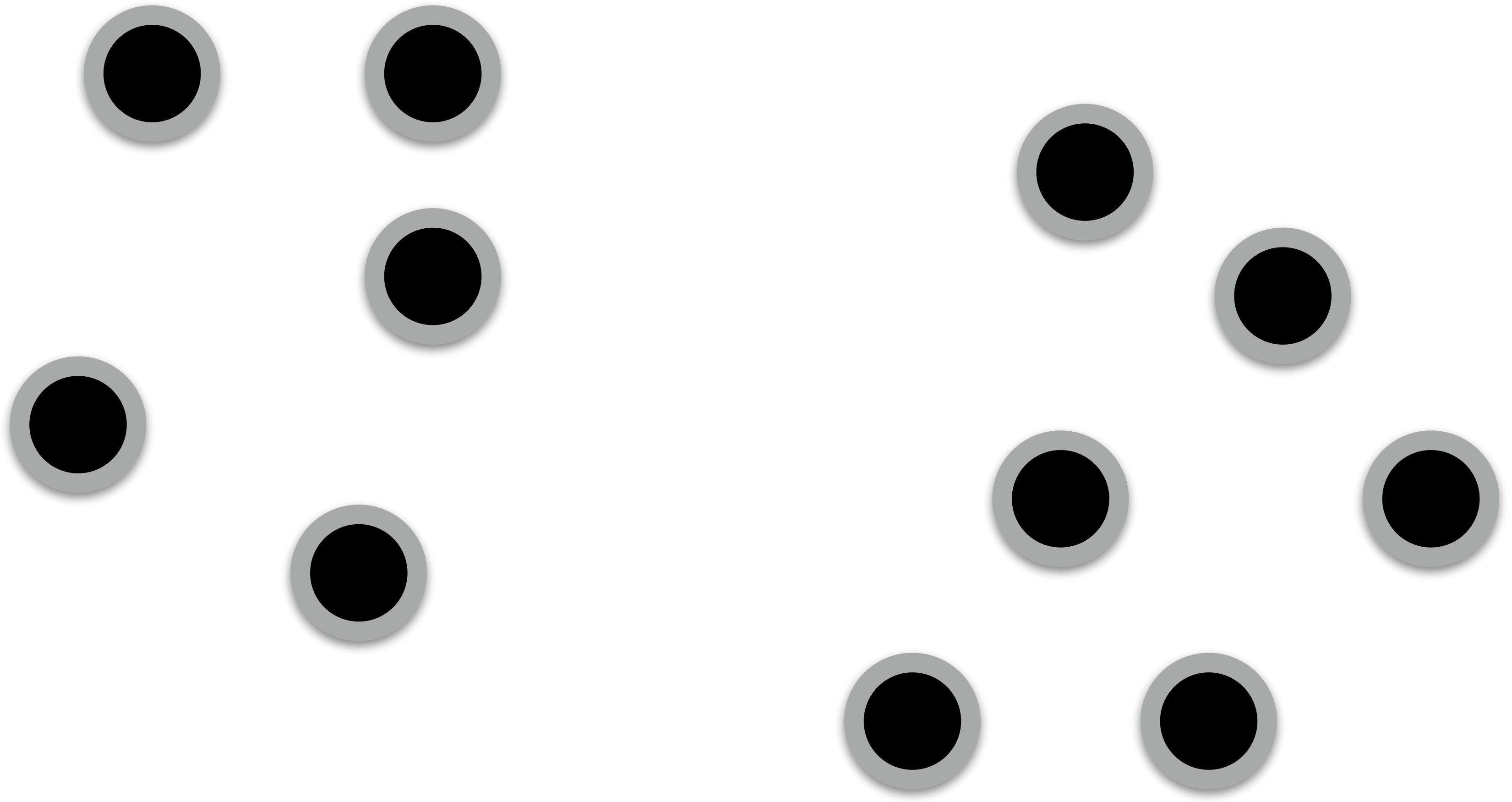
k-Means Clustering / Lloyd's Algorithm

Randomly initialize k cluster centers

Repeat until assignments / centers do not change:

Assign each point to the closest center

Recompute centers as mean of all points assigned to centers



[Lloyd, Least square quantization in PCM's, Bell Telephone Laboratories Paper 1957]
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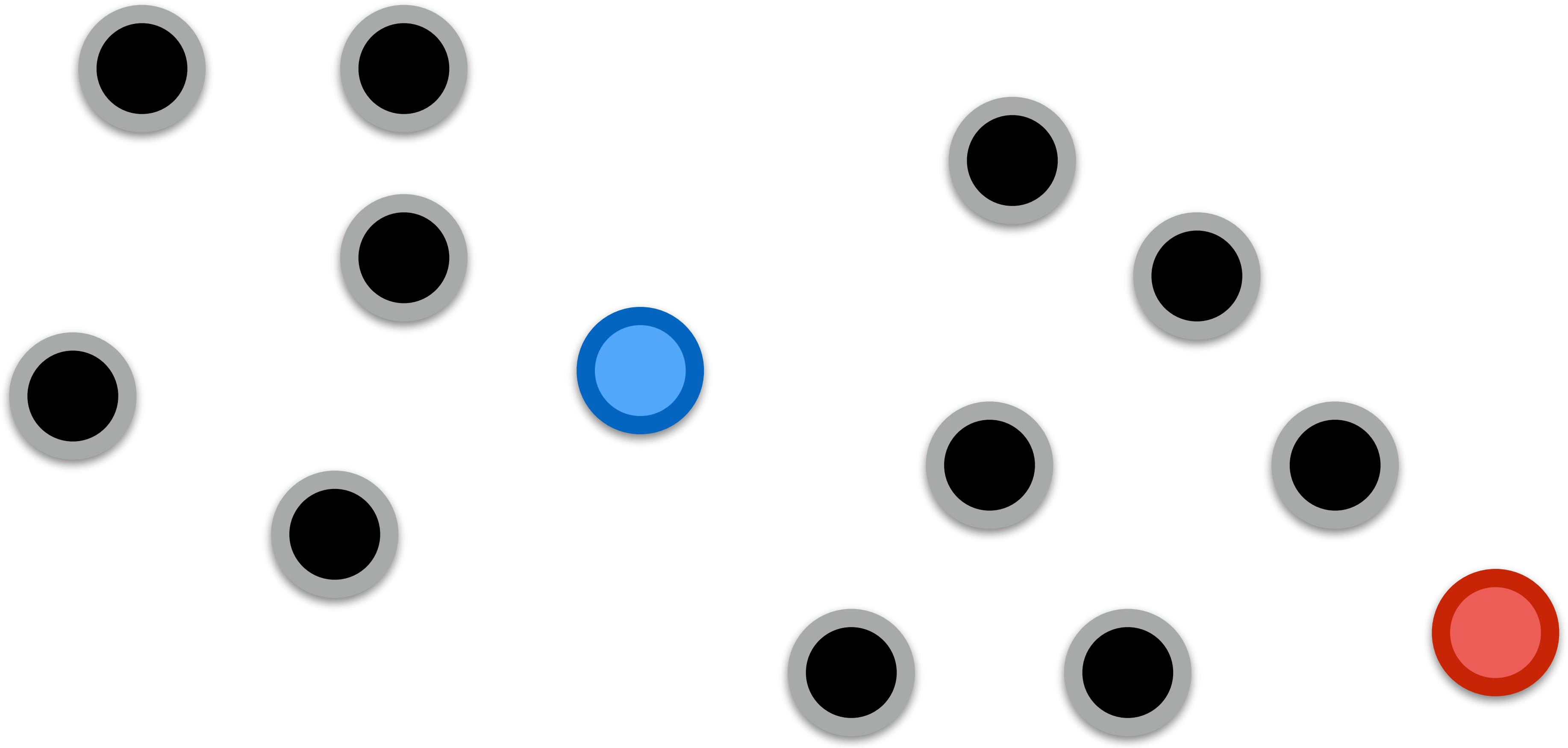

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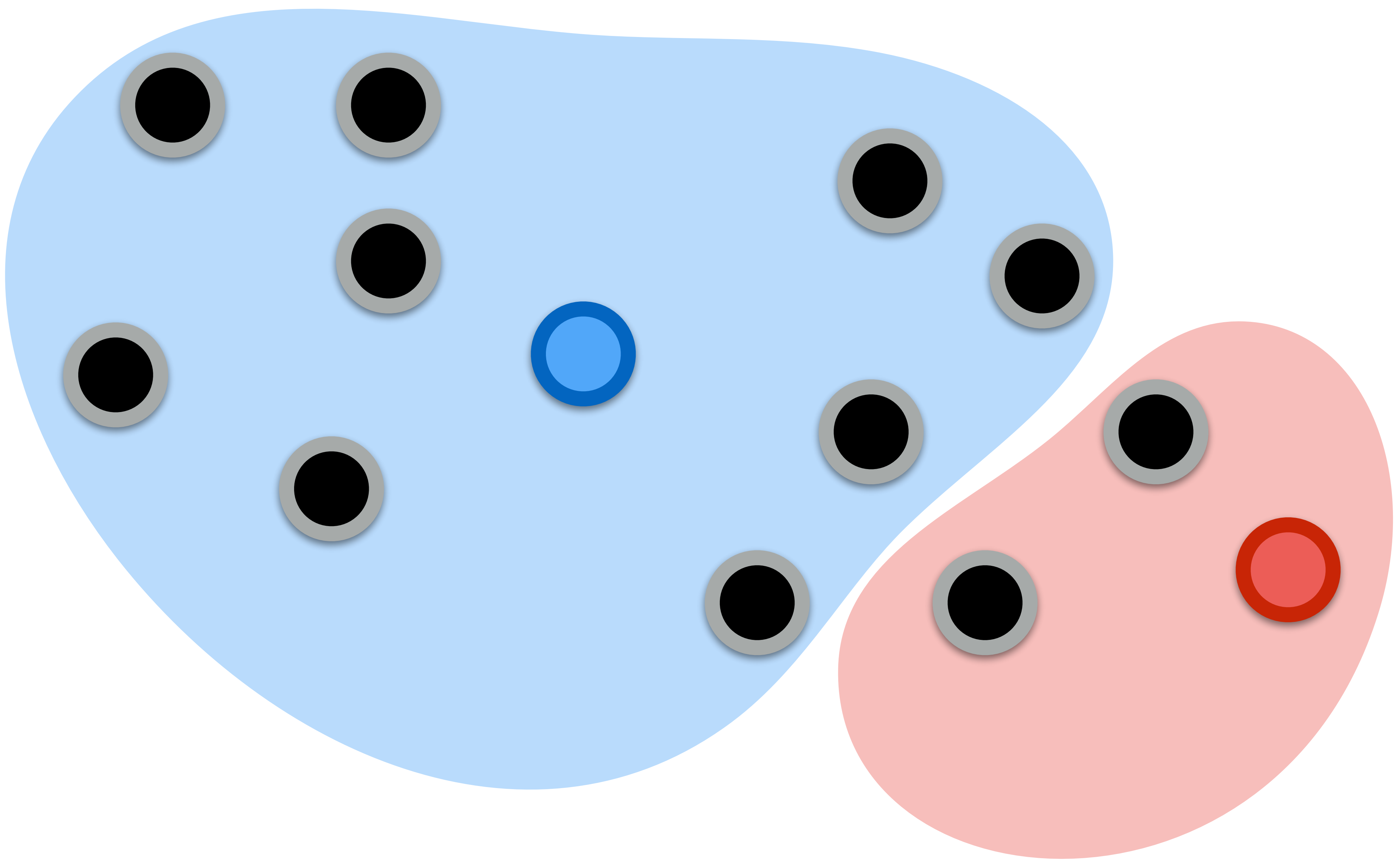

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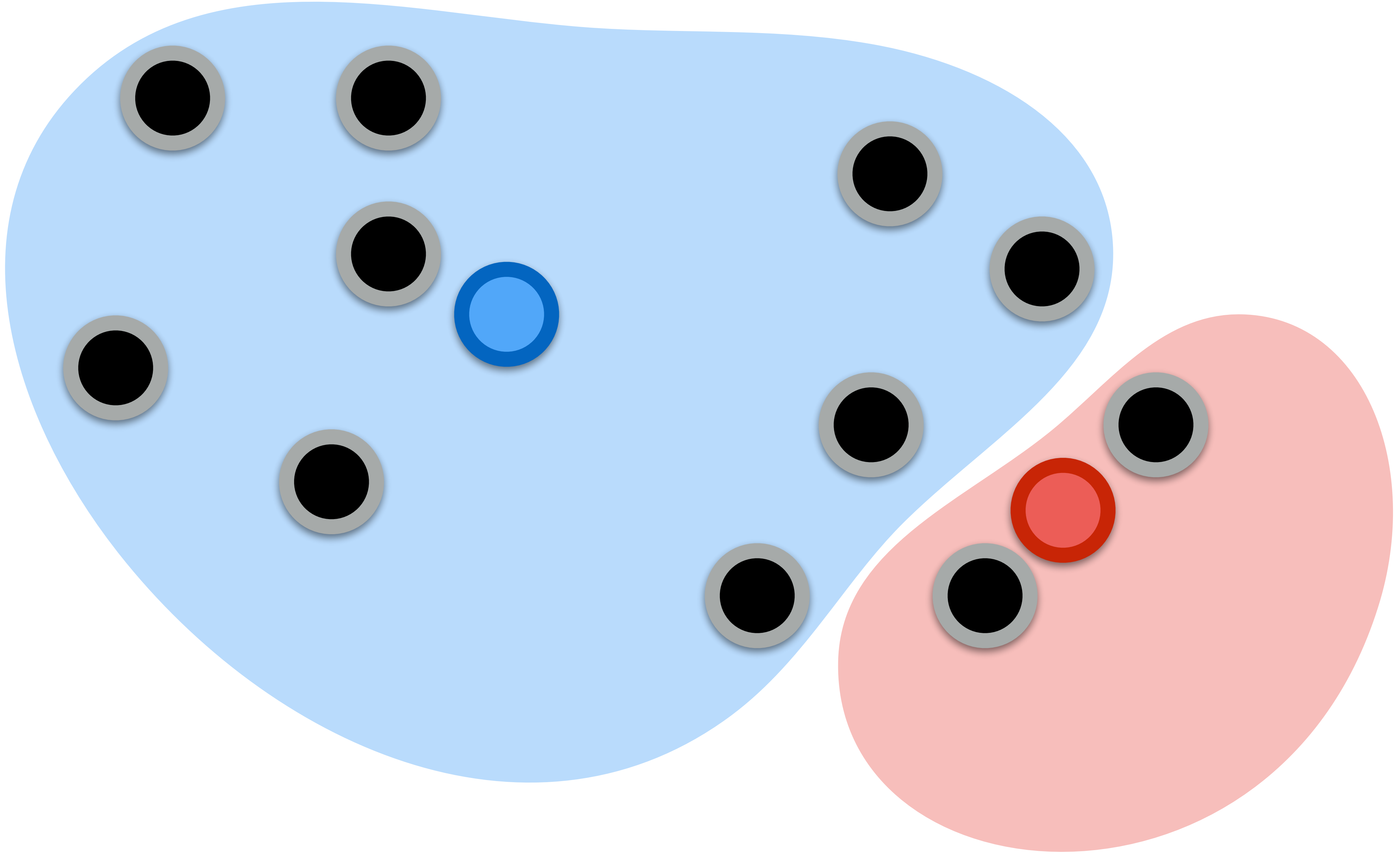

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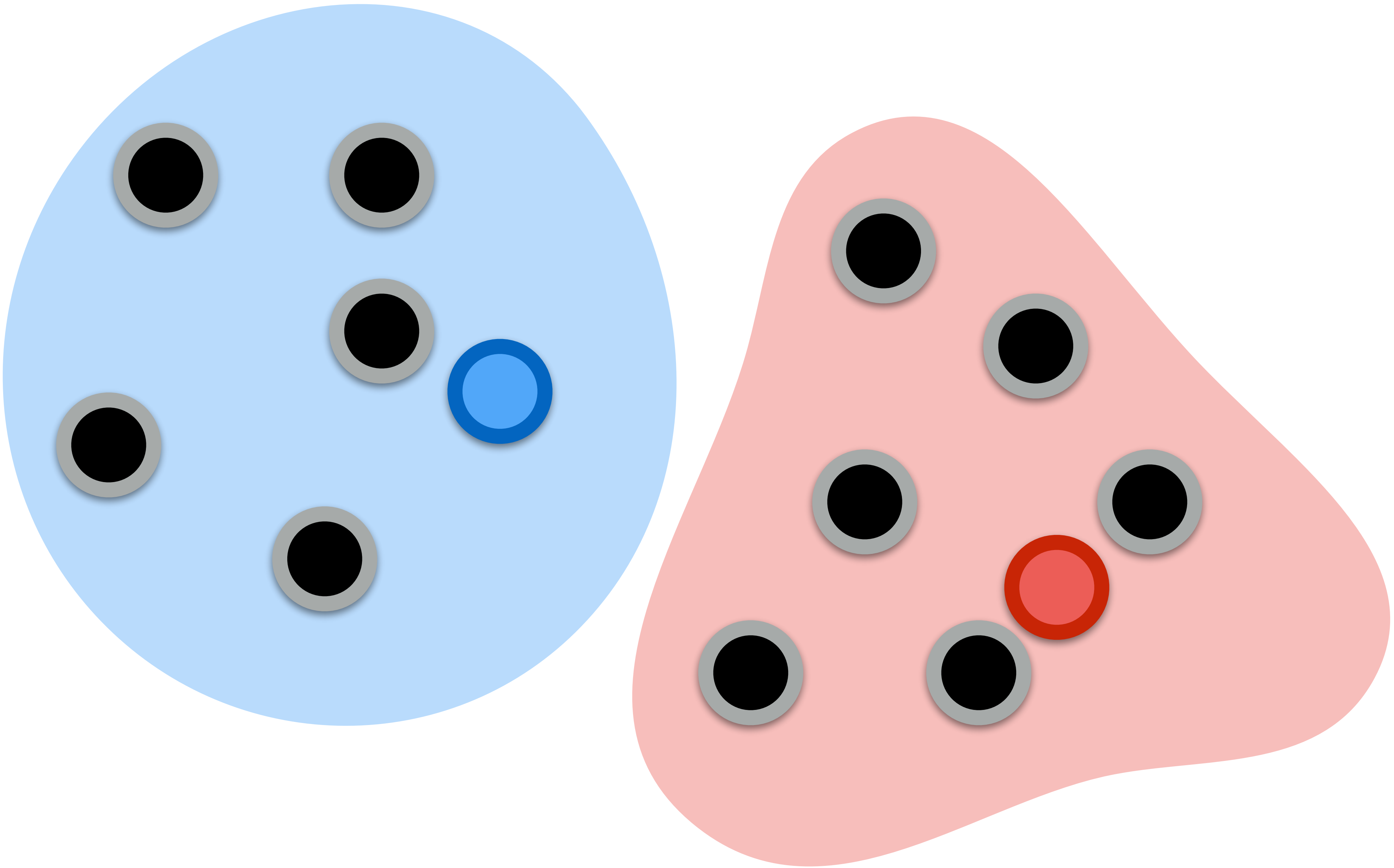

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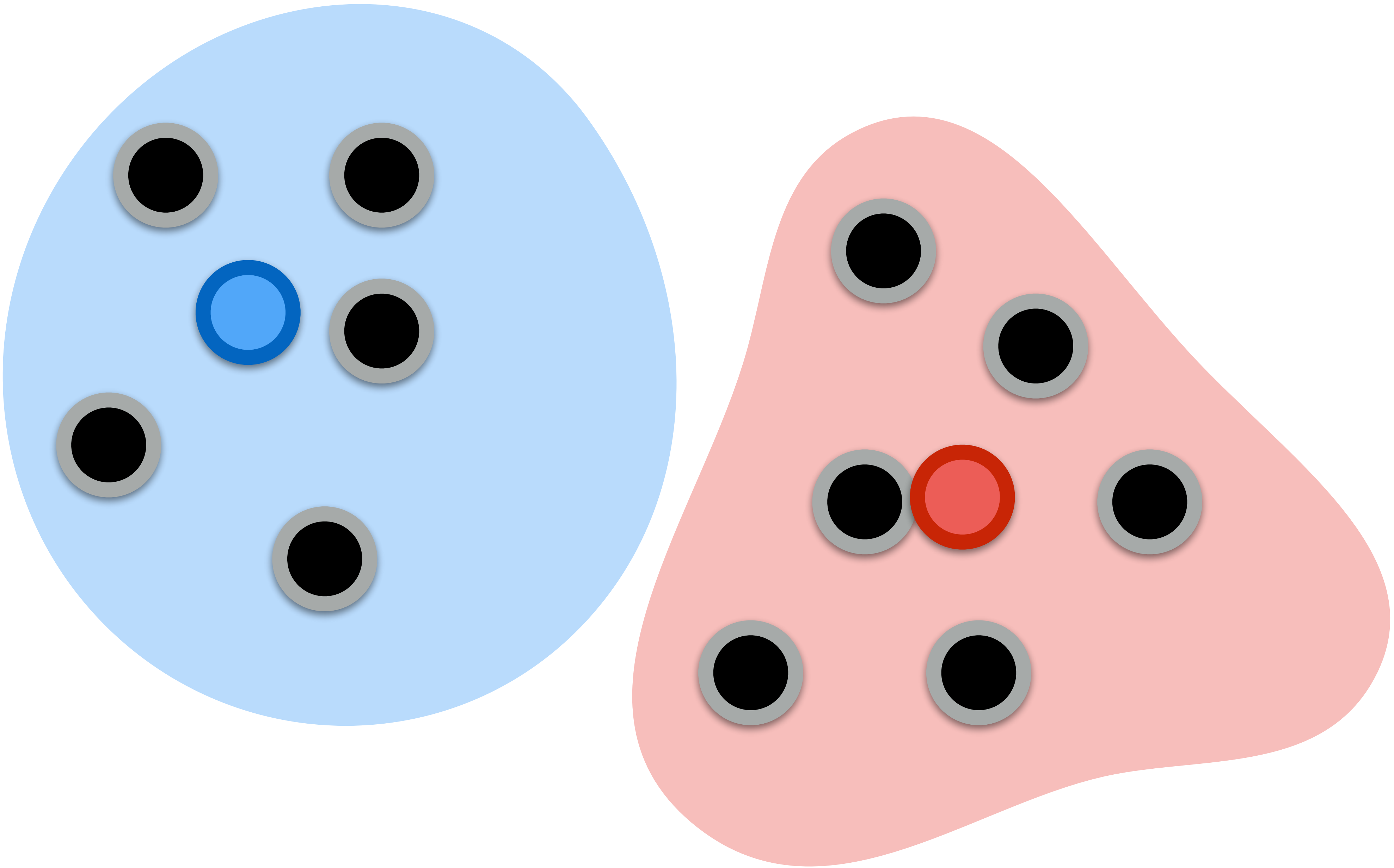

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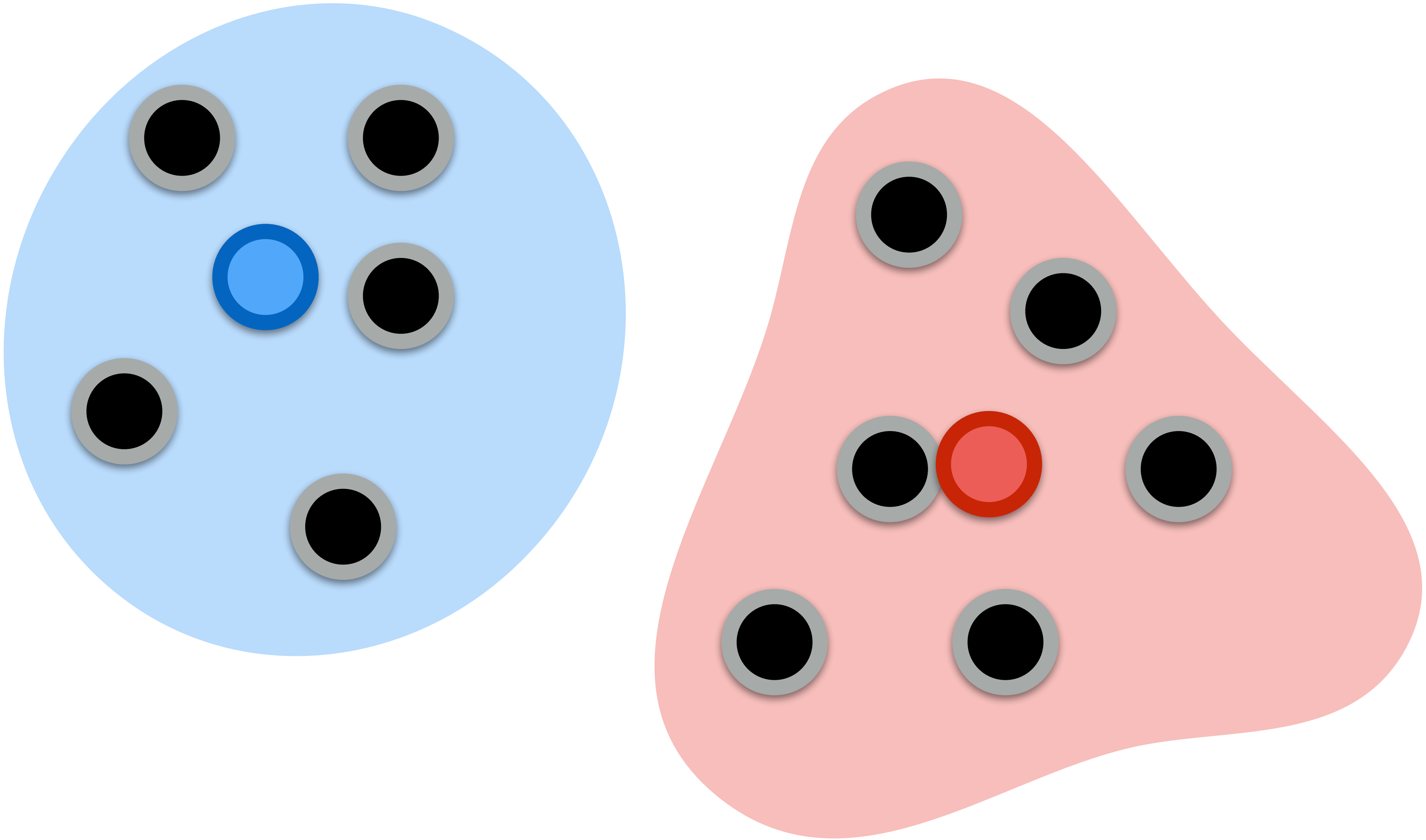

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k-Means Clustering / Lloyd's Algorithm

slide credit: Václav Hlaváč, Bastian Leibe

k-Means Clustering / Lloyd's Algorithm

- Does k-means clustering converge?

slide credit: Václav Hlaváč, Bastian Leibe

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- Finding global optimum is NP-hard even for $k = 2$

slide credit: Václav Hlaváč, Bastian Leibe

k-Means++

- Can we avoid arbitrarily bad local minima?

slide credit: Bastian Leibe, Steve Seitz

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- **k-means++ initialization**:
  - Randomly choose first center
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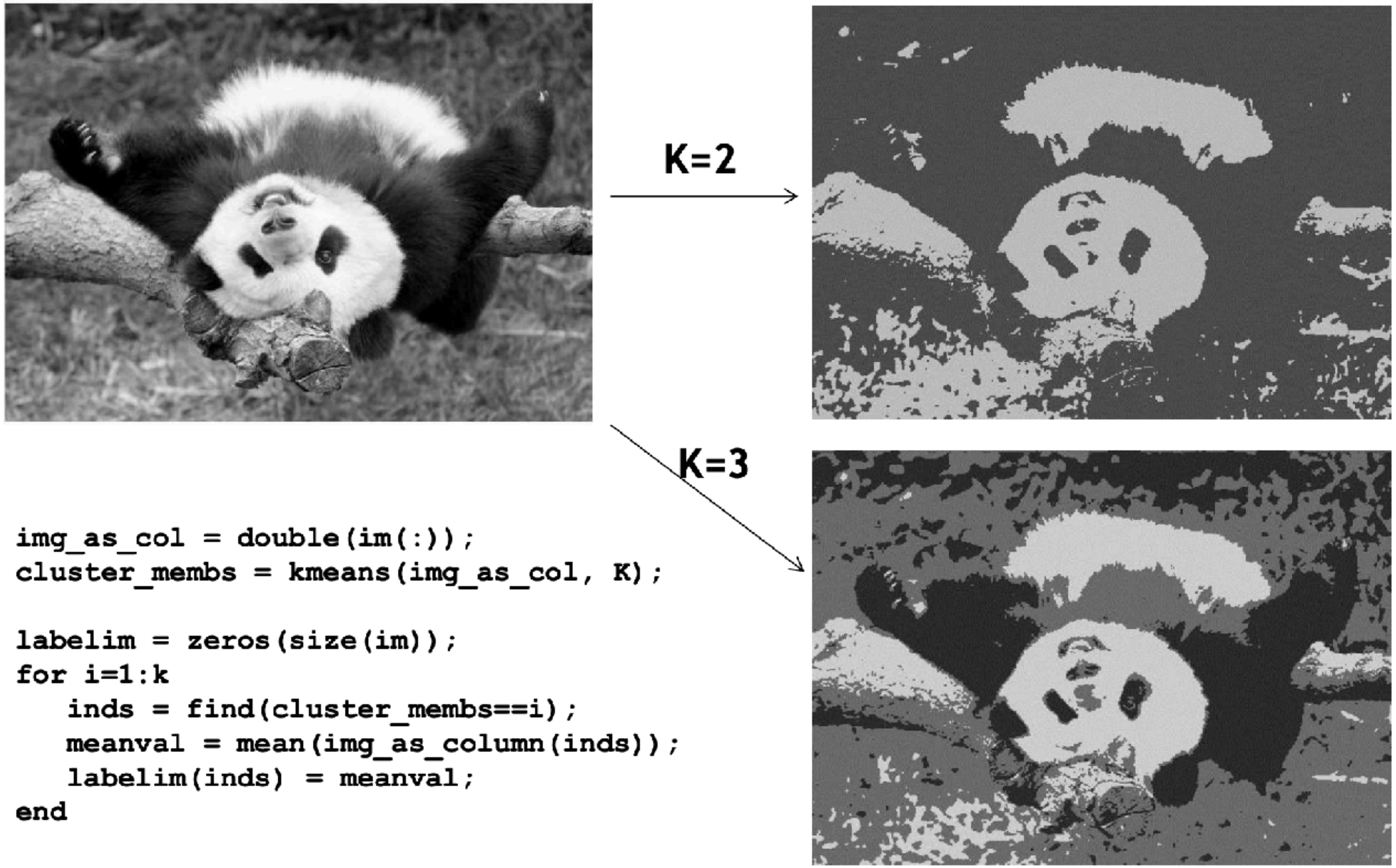
- Arthur & Vassilvitskii 2007: Expected error: $\Theta(\log k) \cdot$ optimum

slide credit: Bastian Leibe, Steve Seitz

Example




```
img_as_col = double(im(:));
cluster_membs = kmeans(img_as_col, K);

labelim = zeros(size(im));
for i=1:k
   inds = find(cluster_membs==i);
   meanval = mean(img_as_column(inds));
   labelim(inds) = meanval;
end
```

slide credit: Bastian Leibe

Example

Image, courtesy Ondoej Drbohlav

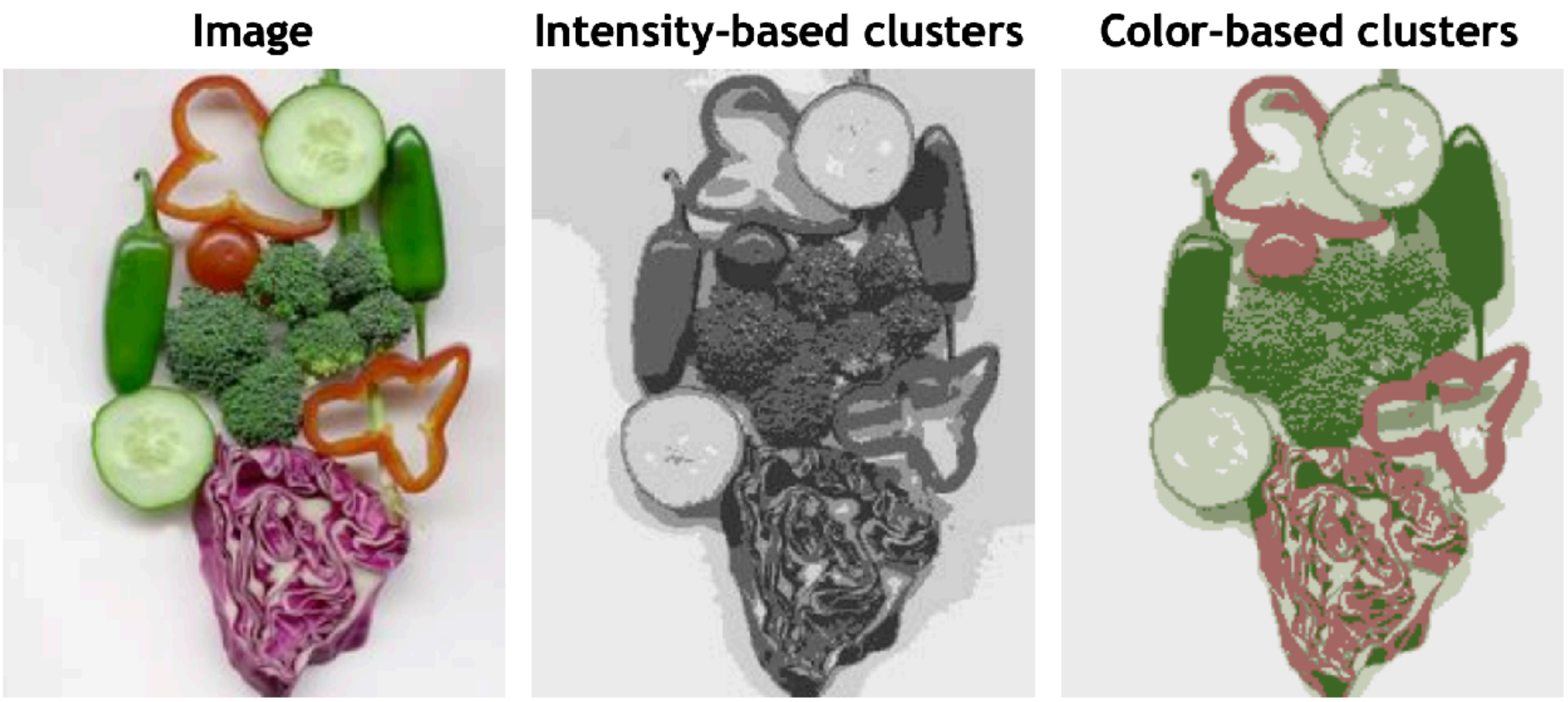
- Clustering into $k = 2$ regions based on absolute values of 1st derivative

$$\left(\left| \frac{\partial I(x,y)}{\partial x} \right|, \left| \frac{\partial I(x,y)}{\partial y} \right| \right)$$

slide credit: Václav Hlaváč

Accounting For Spatial Coherence

- Clustering based on image intensities, color, texture similarity, etc. does not take spatial coherence into account



slide credit: Bastian Leibe, Svetlana Lazebnik

image source: D. Forsyth, J. Ponce, Computer Vision - A Modern Approach, 2nd edition, Pearson, 2011

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- Cluster based on, e.g., color, and pixel position: $(\mathtt{r}, \mathtt{g}, \mathtt{b}, \mathtt{x}, \mathtt{y})$

slide credit: Bastian Leibe, Svetlana Lazebnik

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k-Means Clustering - Discussion

- **Pros**:
  - Simple to implement, fast to compute, many good implementations available (Matlab, Python, …)
  - Converges to local minimum

slide credit: Václav Hlaváč, Bastian Leibe, Kristen Grauman

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  - Mean needs to be defined

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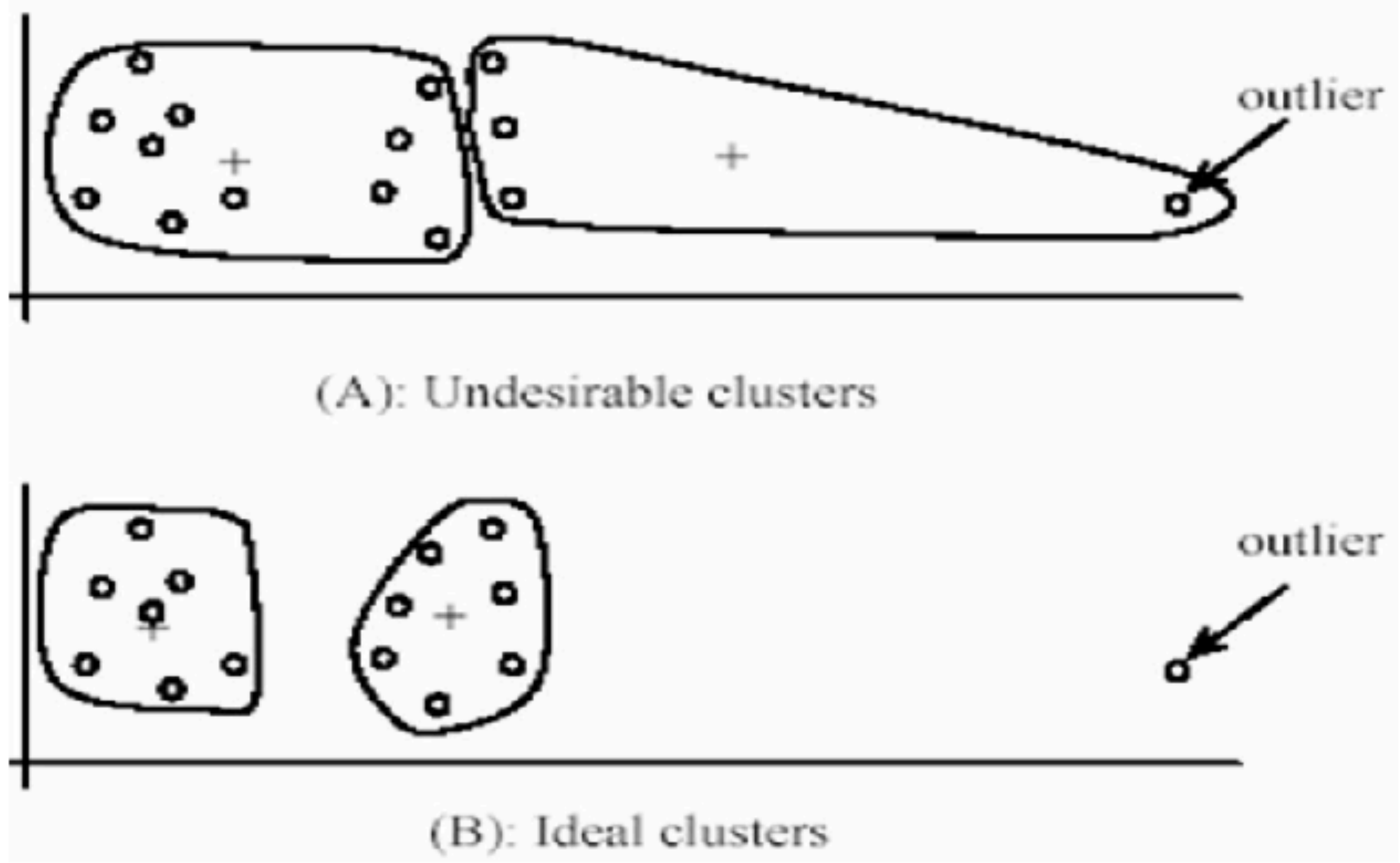

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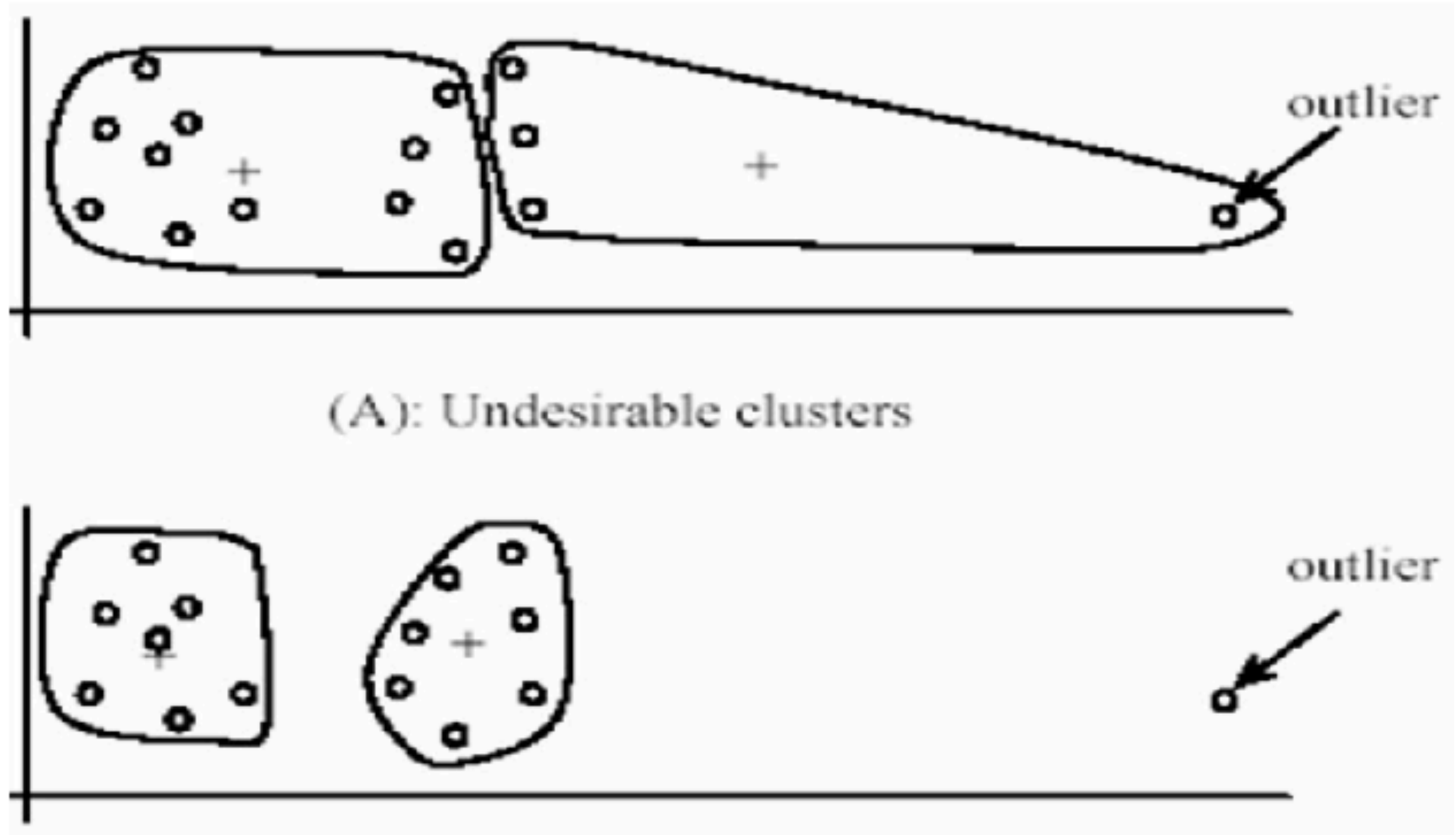
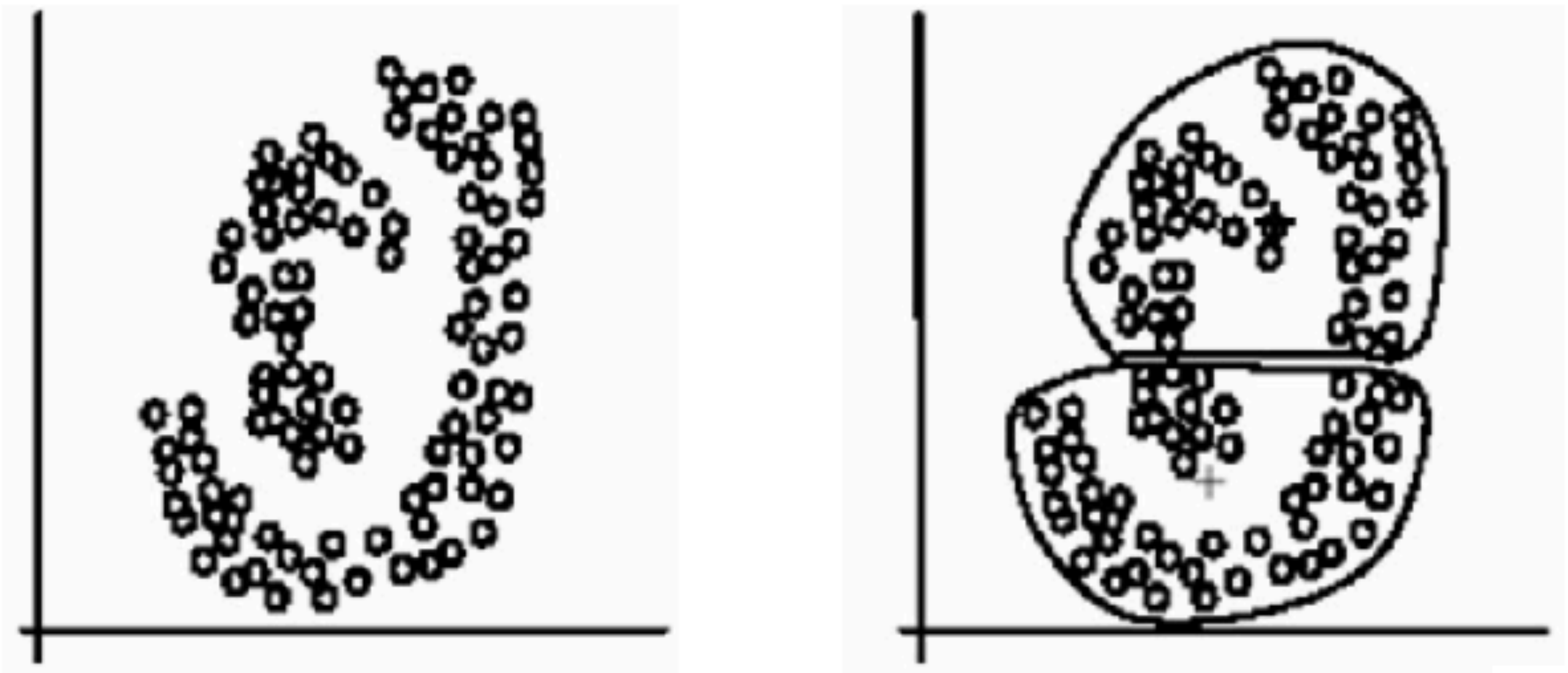
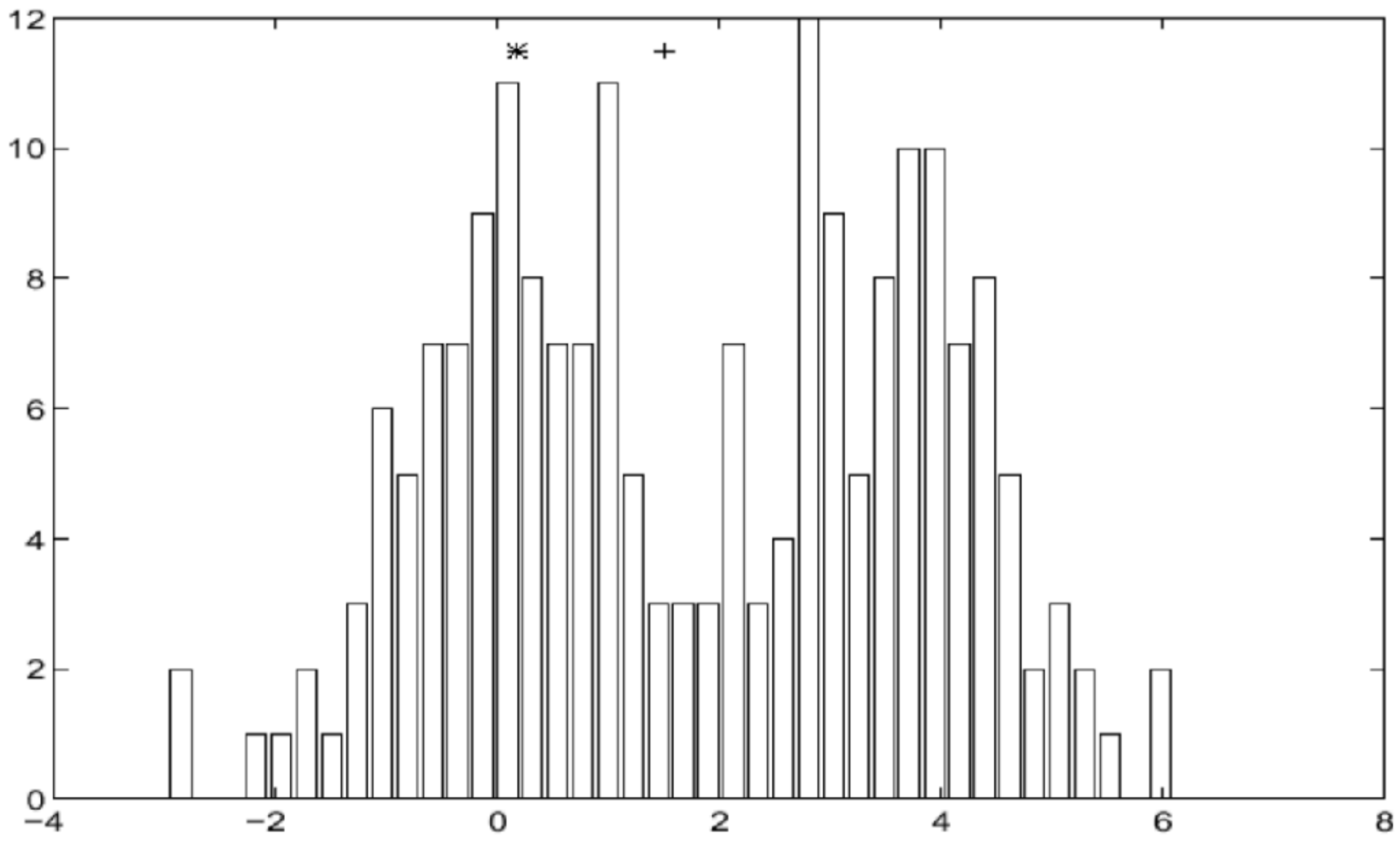


(A): Undesirable clusters
(B): Ideal clusters


(A): Two natural clusters
(B): k-means clusters

slide credit: Václav Hlaváč, Bastian Leibe, Kristen Grauman

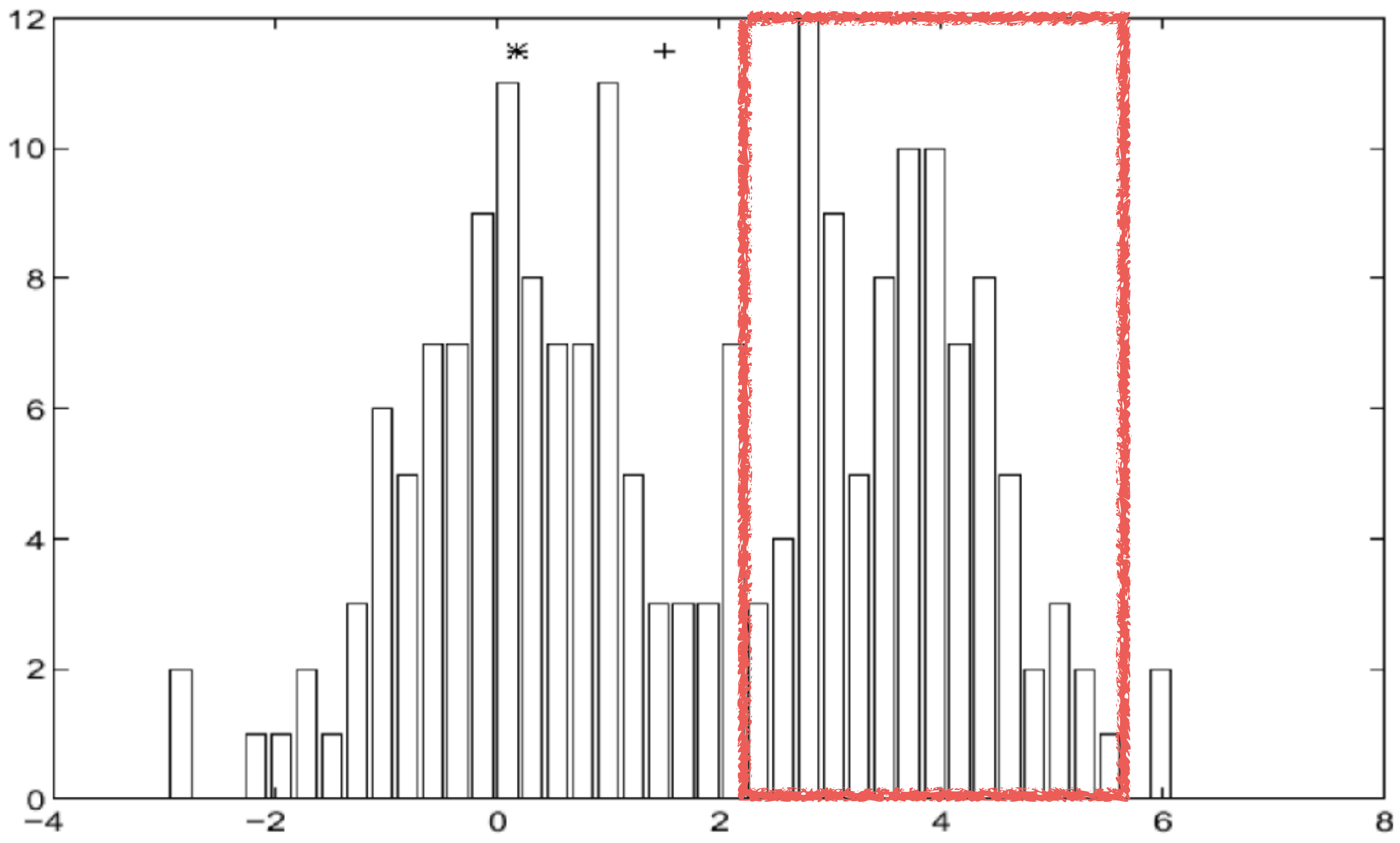
Mean Shift Clustering



- How many modes (= cluster centers) are there?

slide credit: Steve Seitz, Bastian Leibe

Mean Shift Clustering



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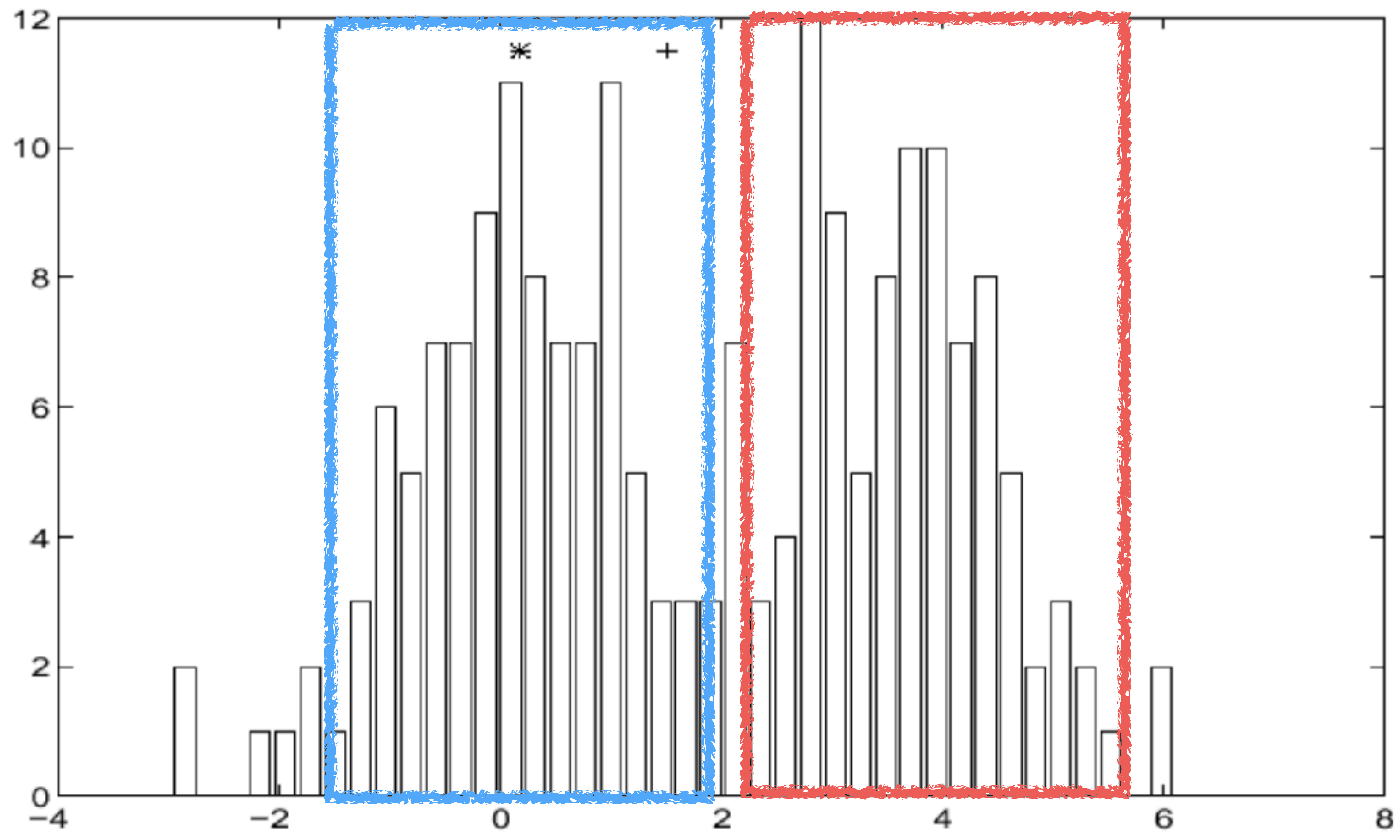

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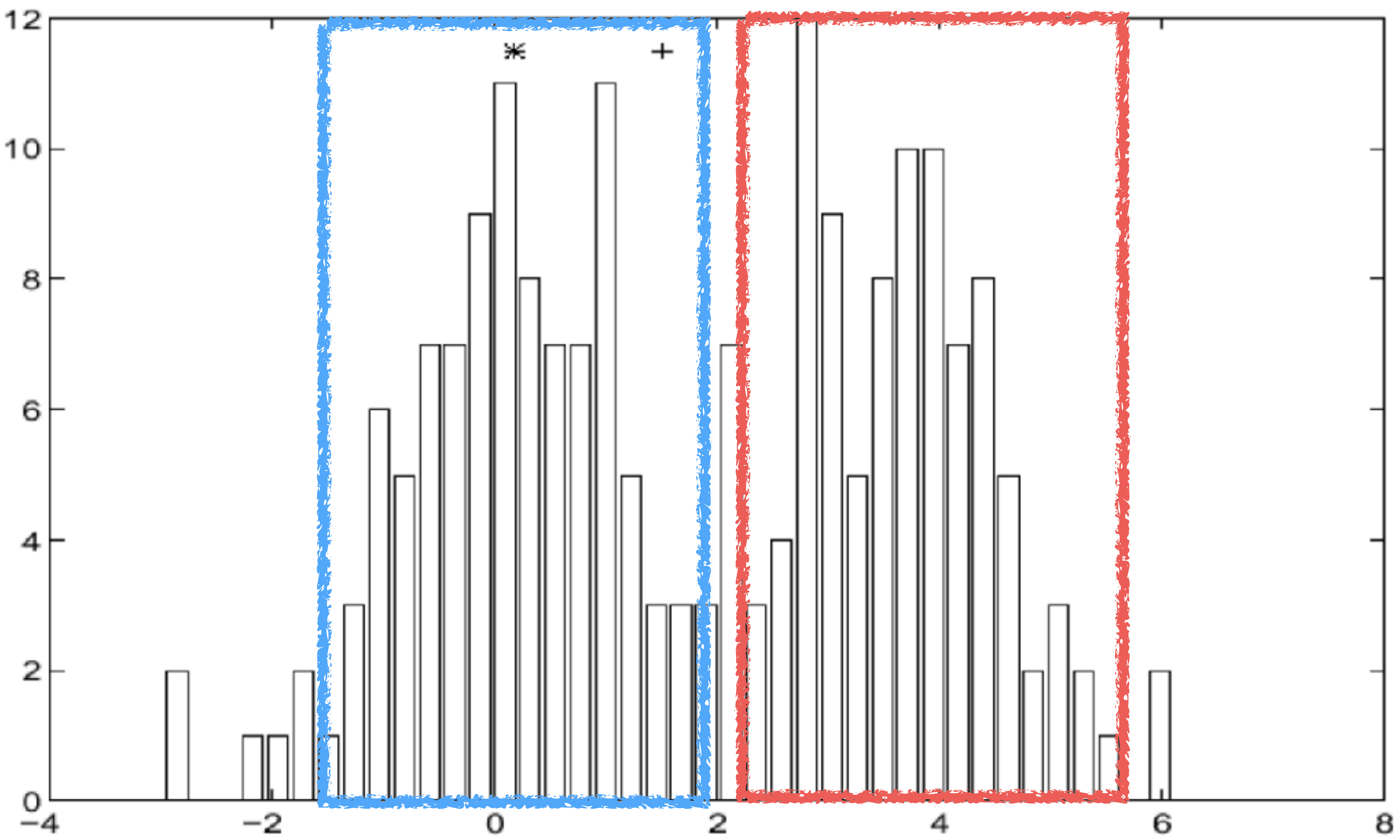

- How many modes (= cluster centers) are there?
- How to detect automatically?

slide credit: Steve Seitz, Bastian Leibe

Mean Shift Clustering

- **Modes** = local maxima of density of given distribution

slide credit: Václav Hlaváč, Bastian Leibe

Mean Shift Clustering

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- Assumption: density increases towards the modes

slide credit: Václav Hlaváč, Bastian Leibe

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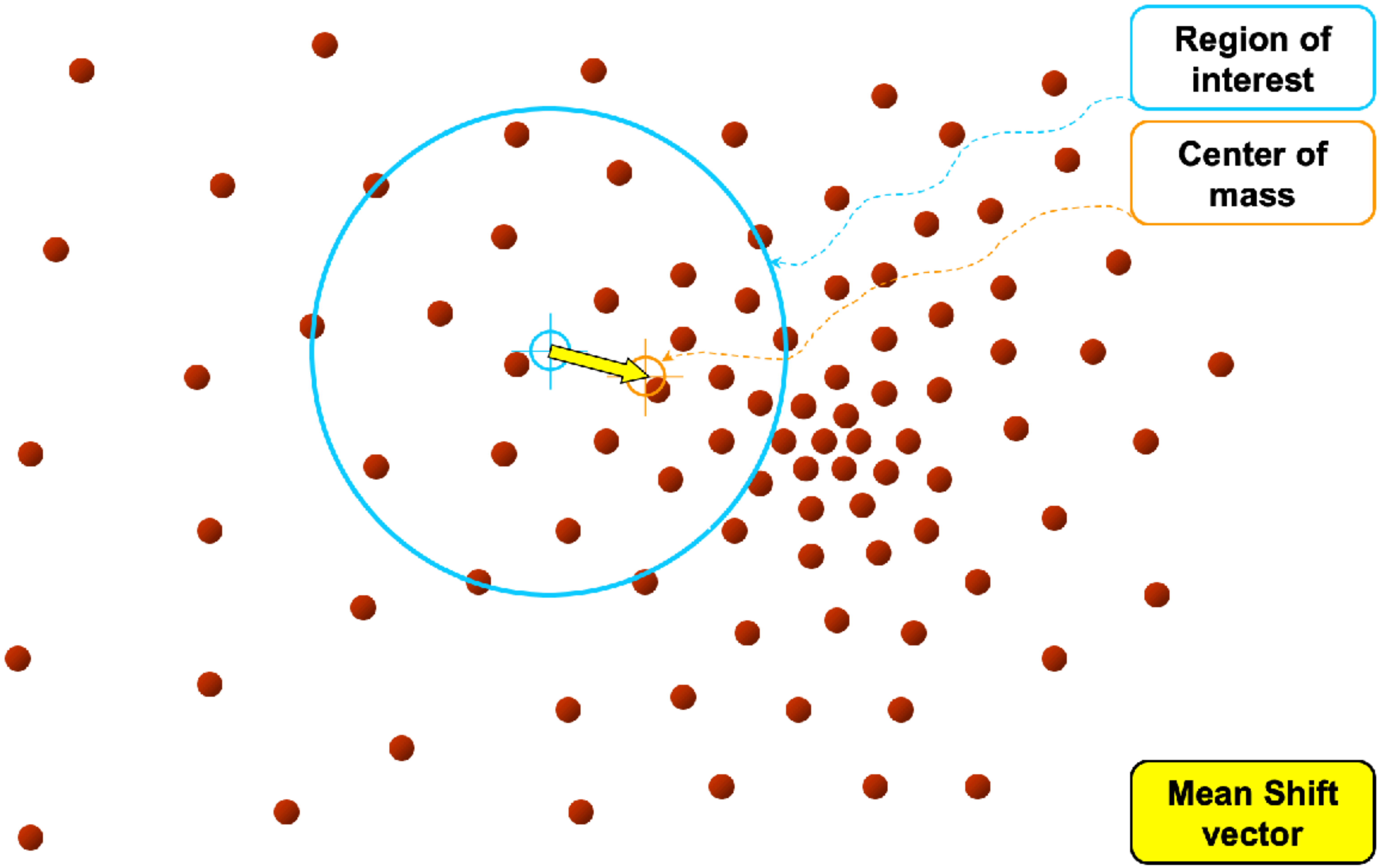
  - Sample **mean of local samples** points in the direction of higher density, provides estimate of the gradient
  - Mean shift vector m for point p:

$$m = \sum_{i \in \text{window}} w_i \left(p_i - p\right), \quad w_i = \text{dist}(p, p_i)$$

slide credit: Václav Hlaváč, Bastian Leibe

Mean Shift Algorithm



slide credit: Y. Ukrainitz & B. Sarel

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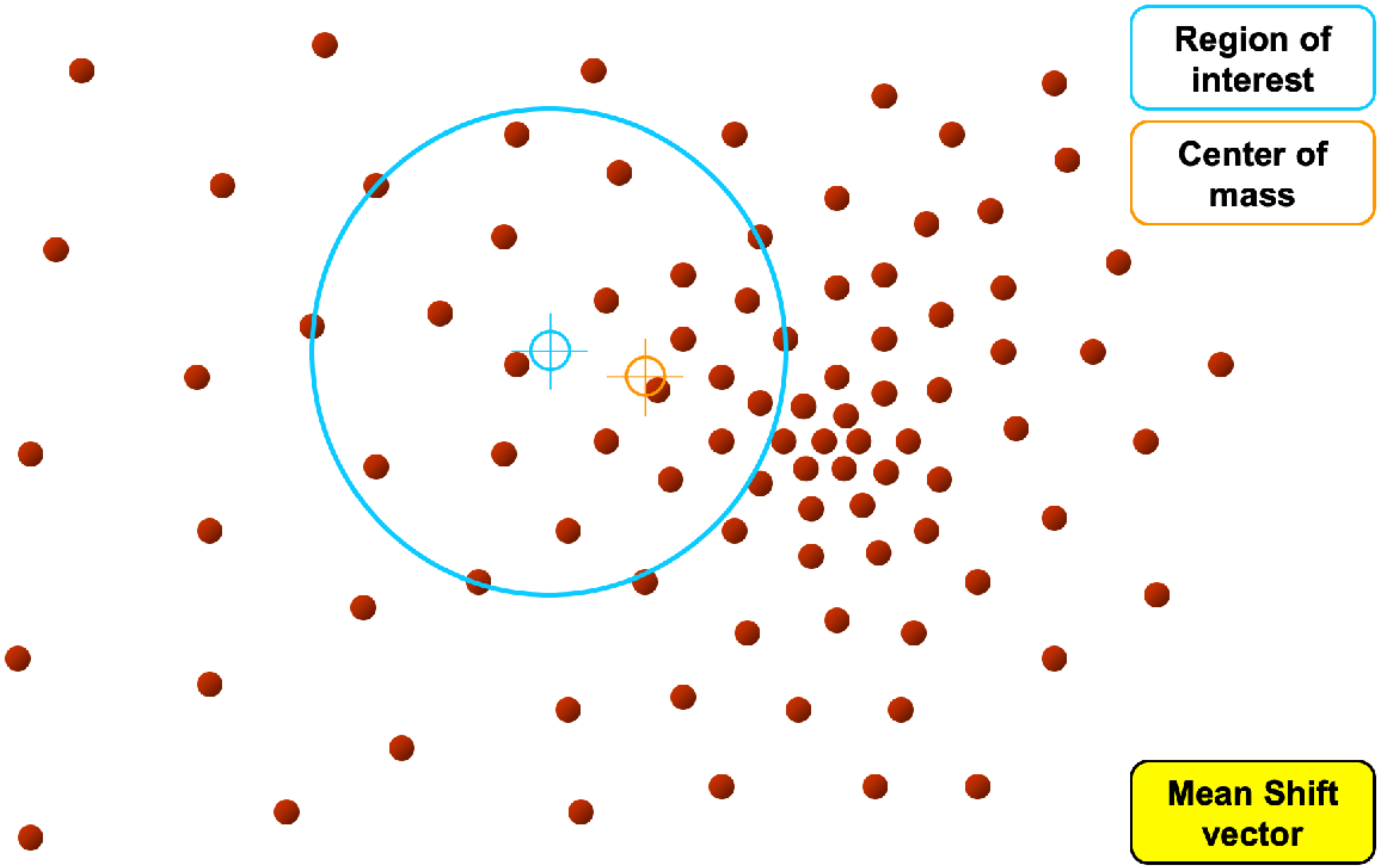

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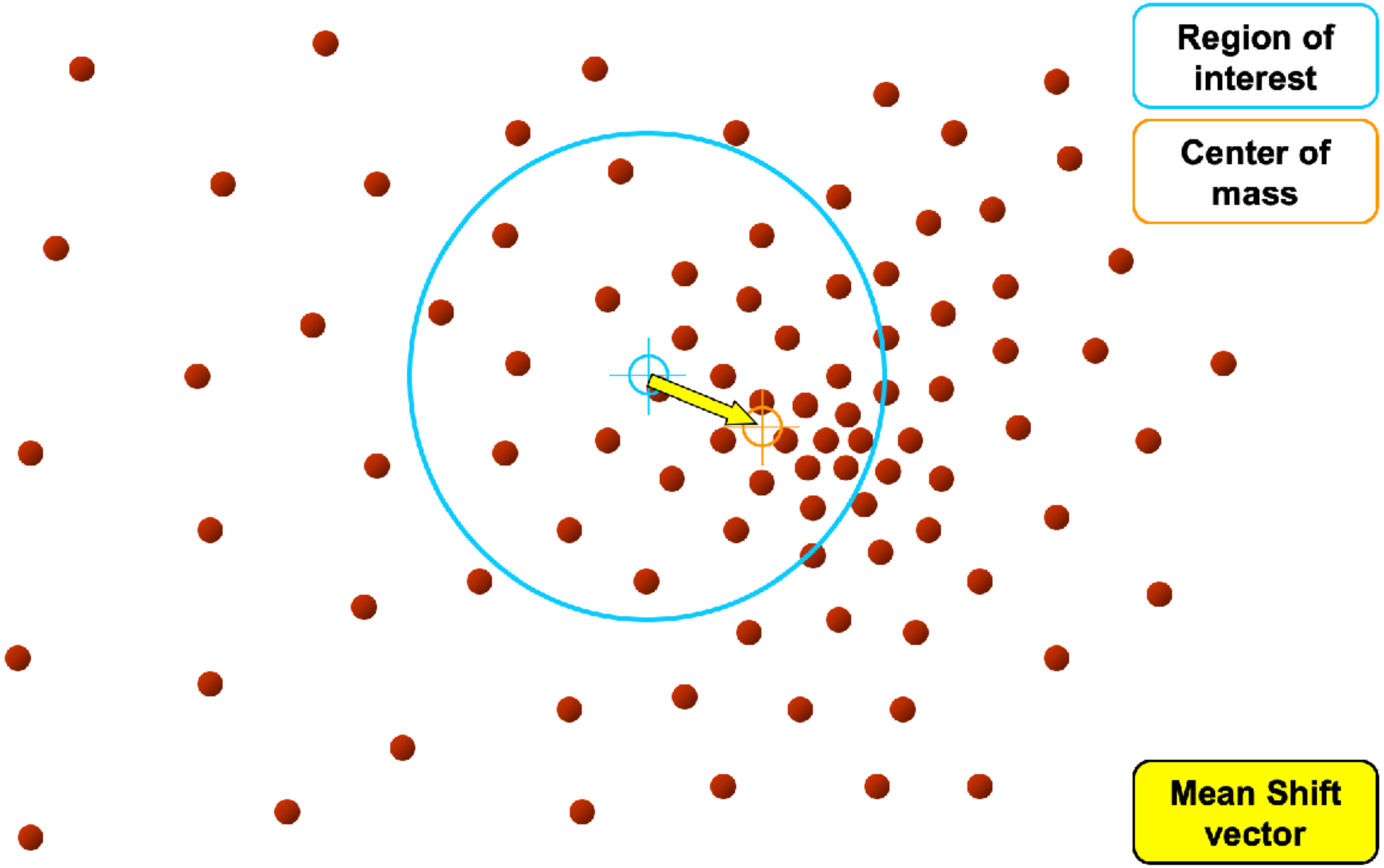

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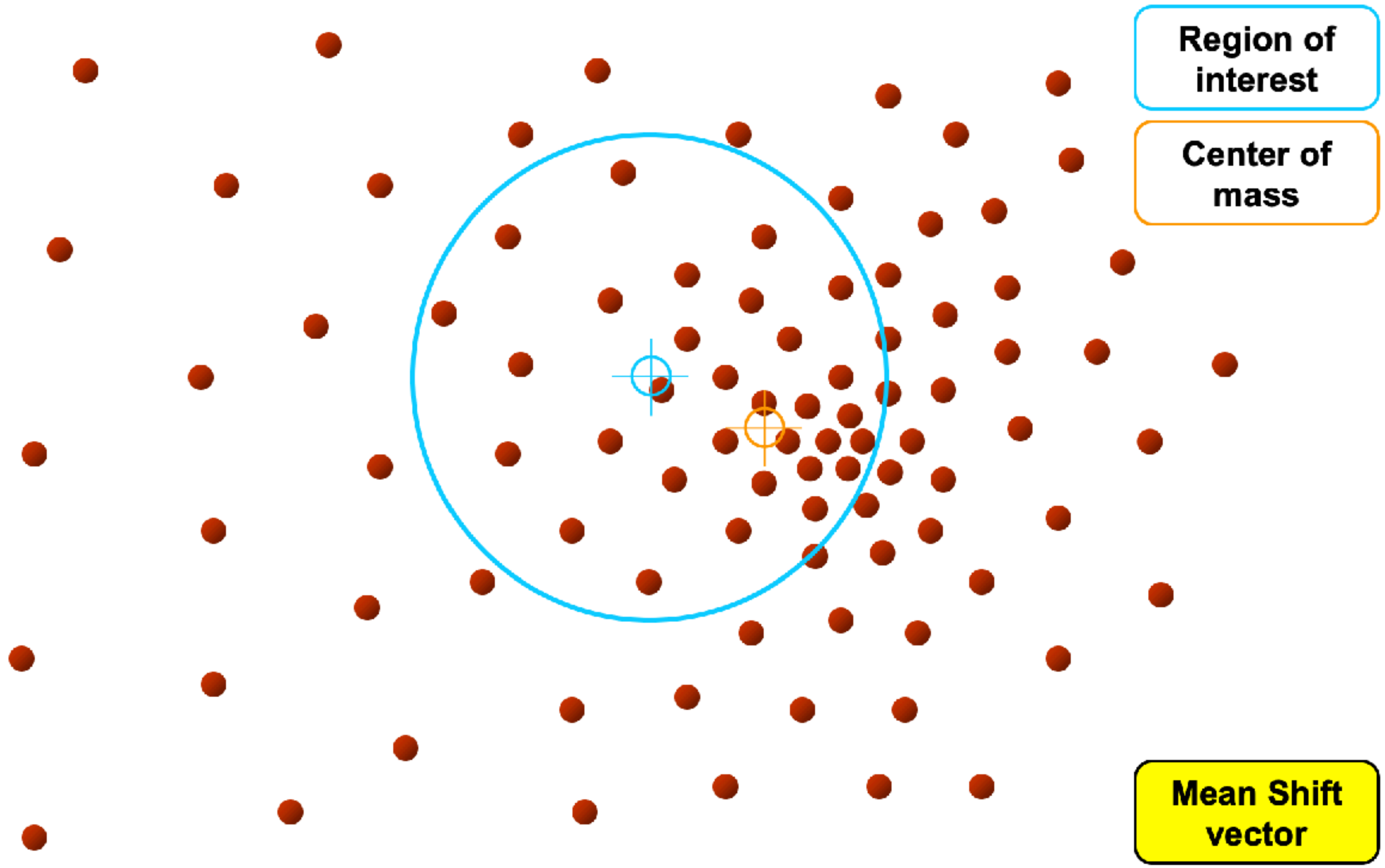

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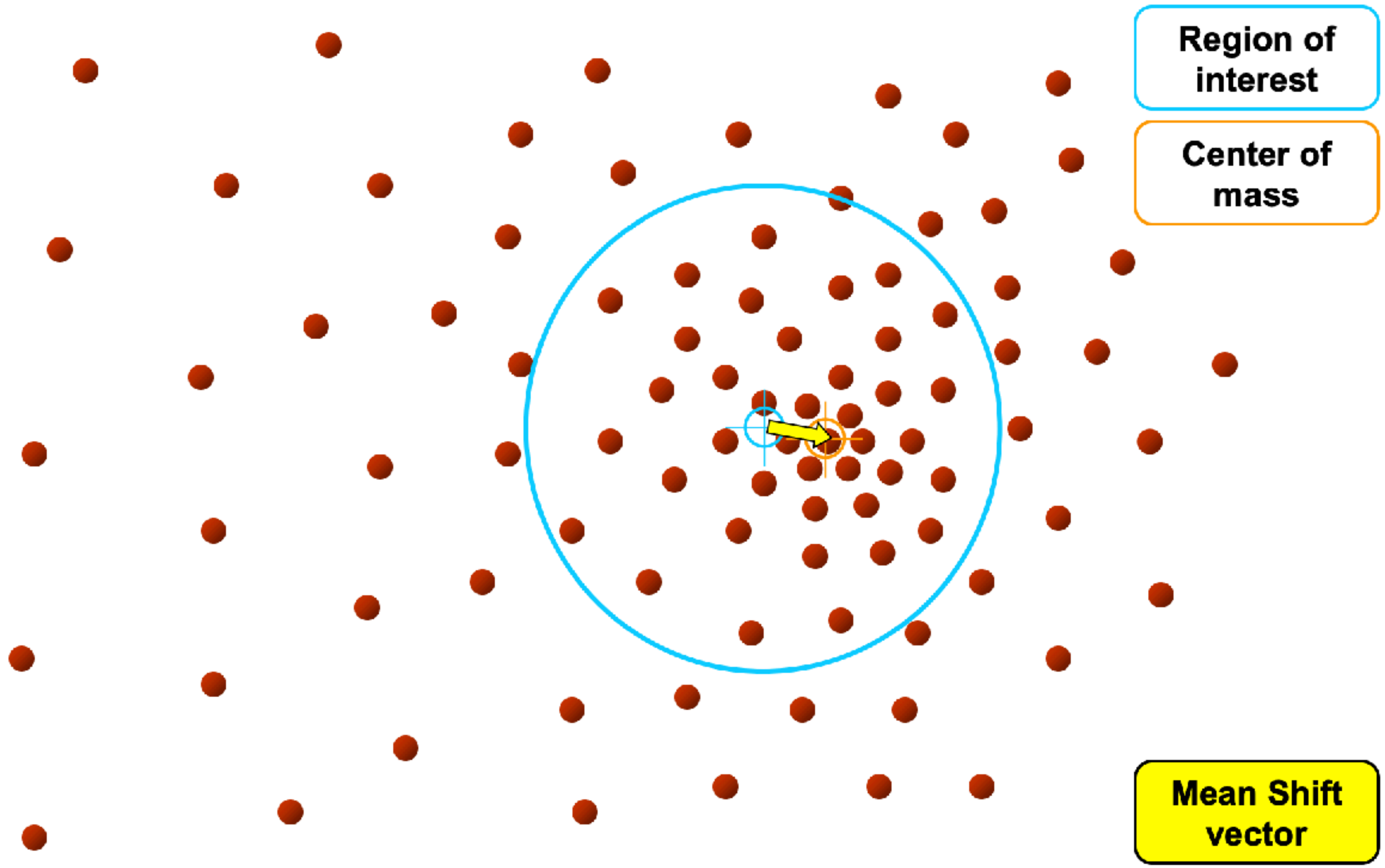

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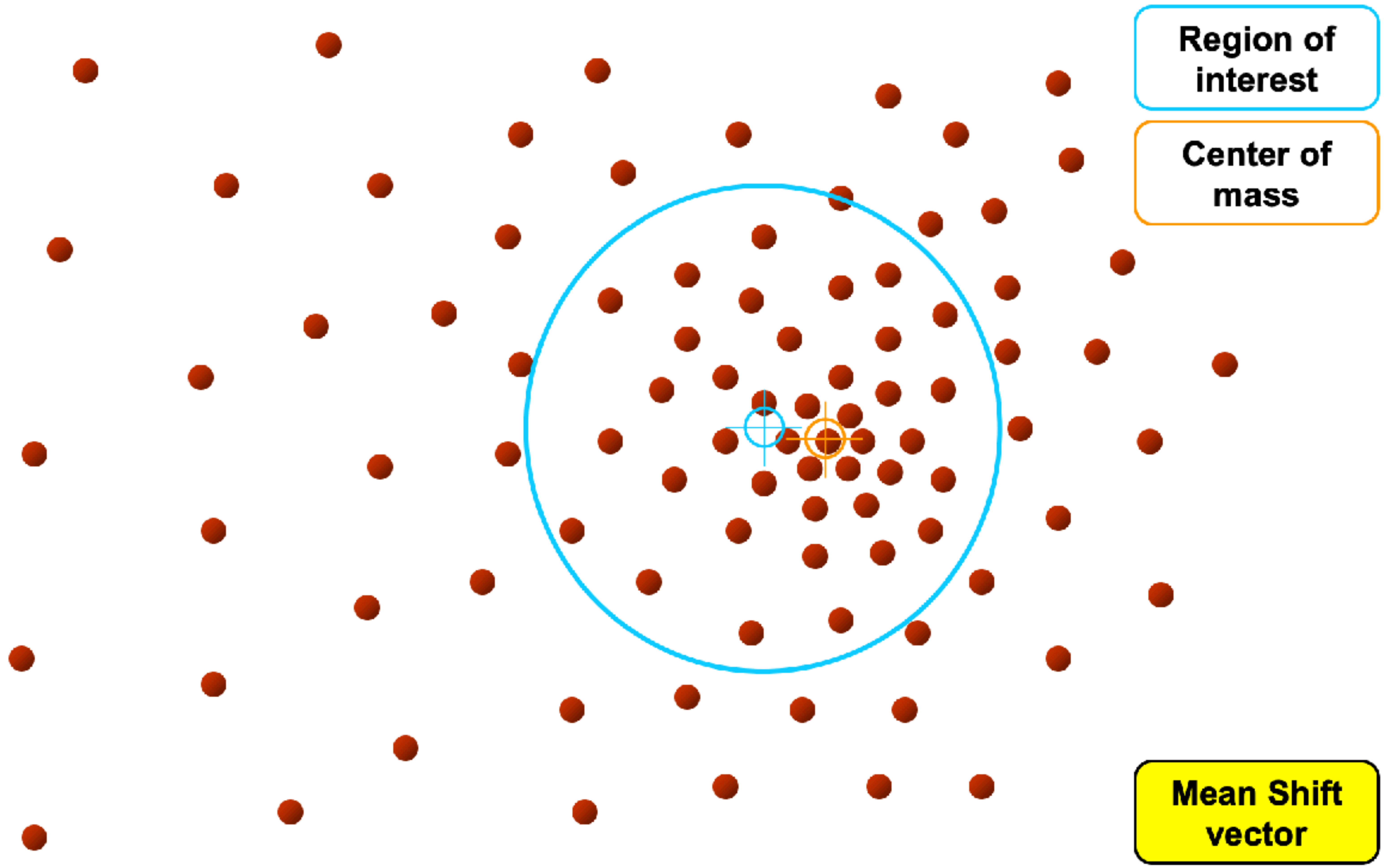

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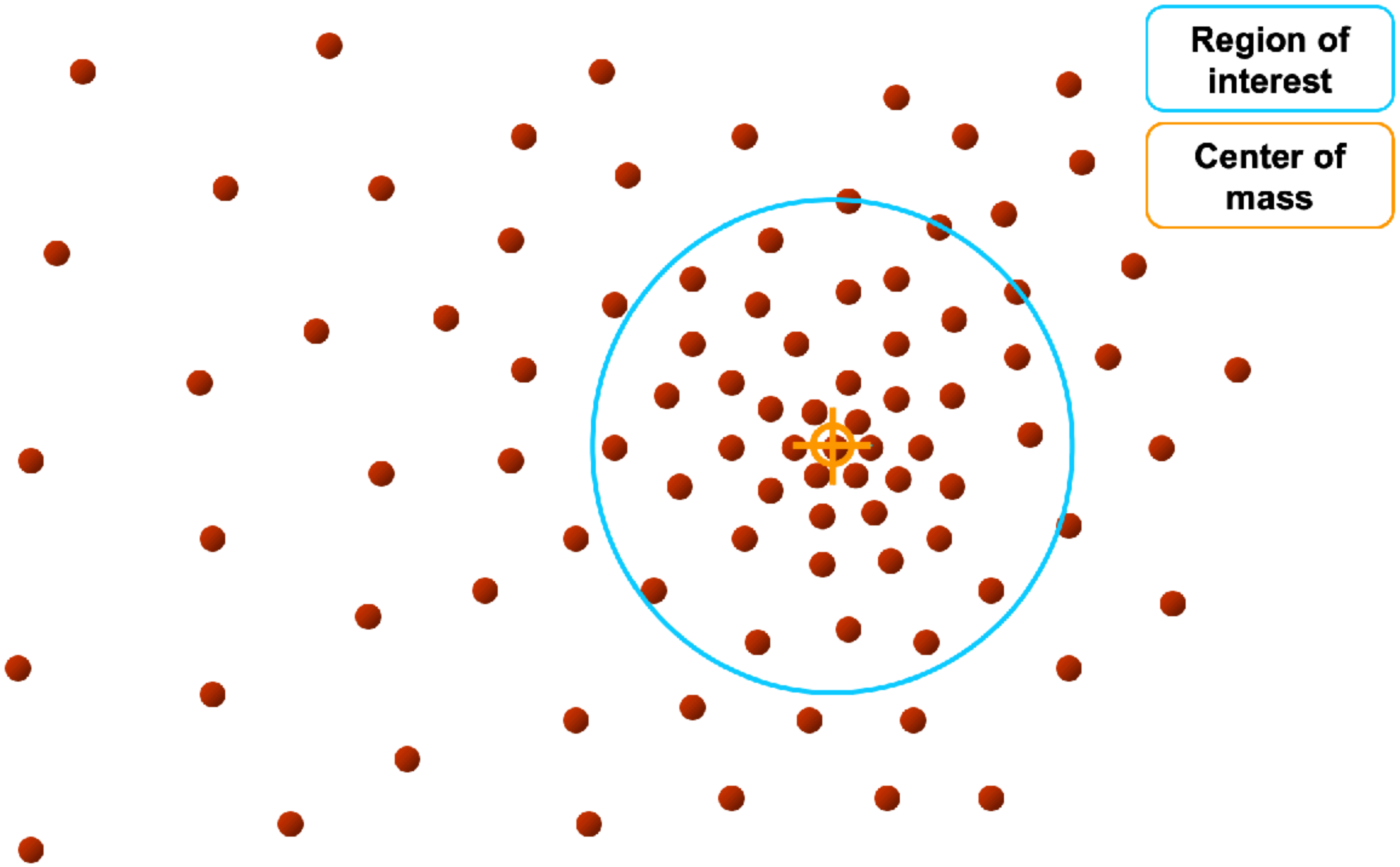

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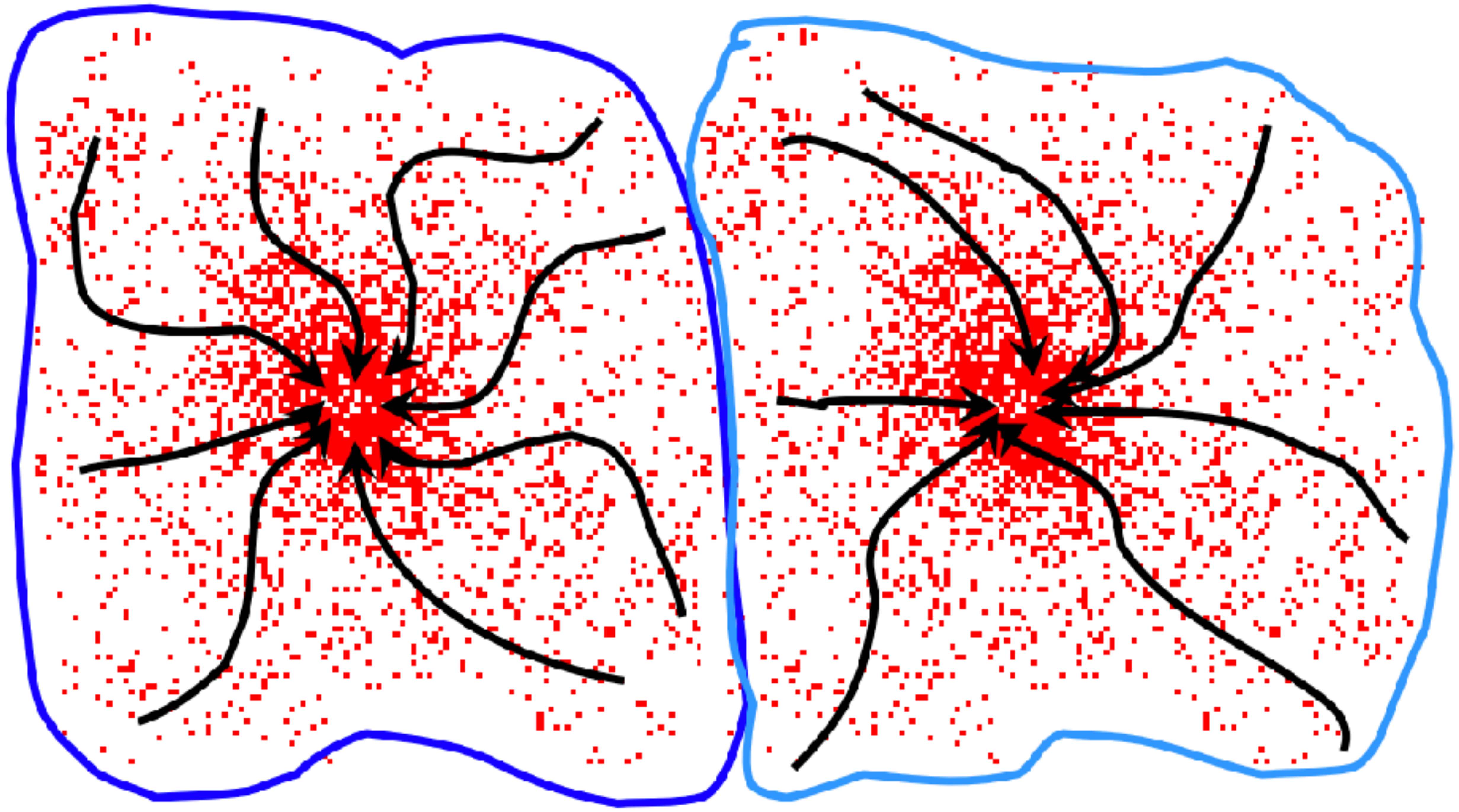
Mean Shift Clustering

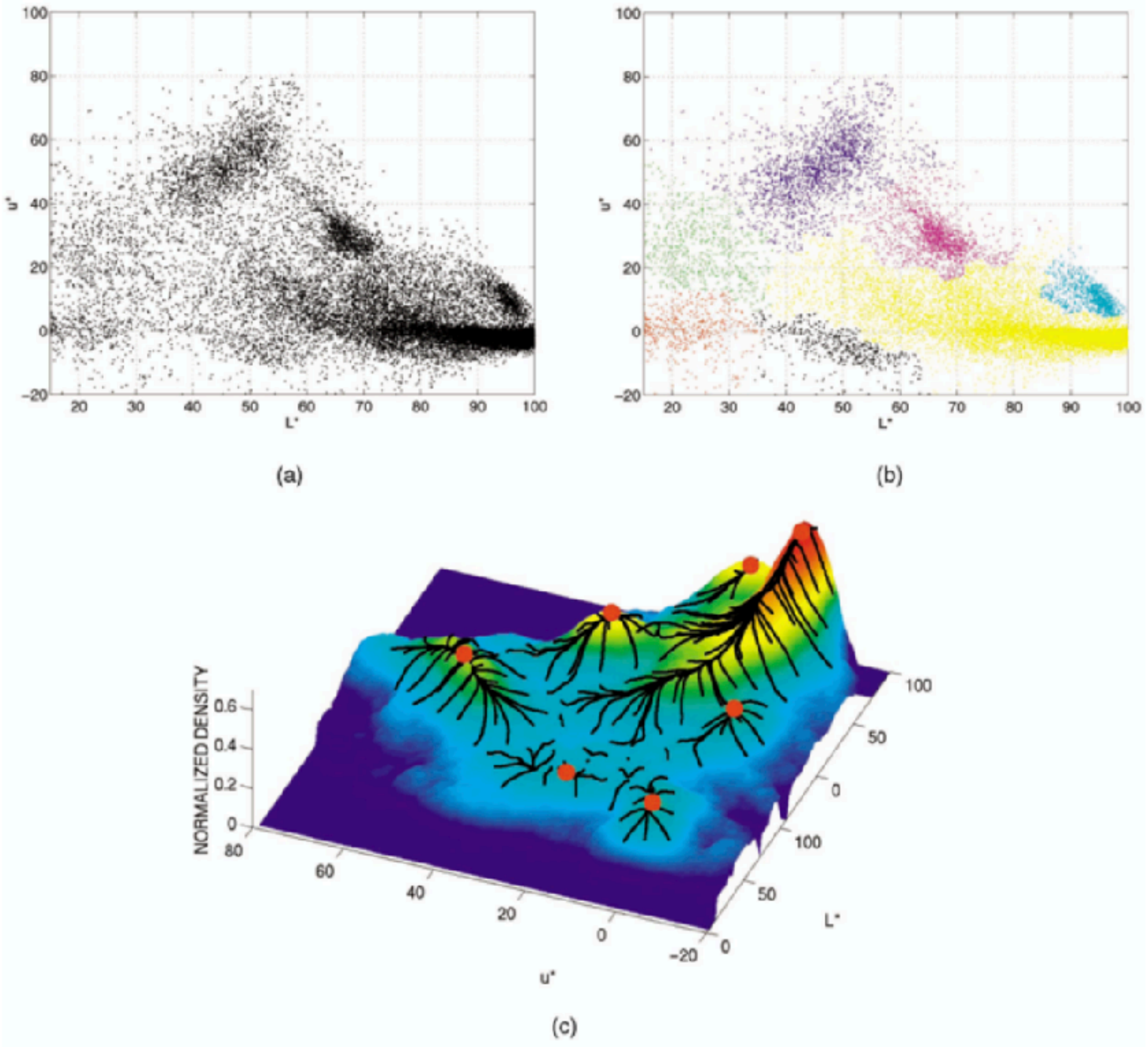
- **Clusters**: all data points in attraction basin of mode
- **Attraction basin**: regions where mean shift leads to same mode



slide credit: Bastian Leibe, Y. Ukrainitz & B. Sarel

Mean Shift Segmentation

- Extract features (colors, intensities, gradients, etc.)
- Initialize search windows at pixel positions / uniformly distributed over image
- Run mean shift for each window
- Merge windows that end up on the same “peak” or mode



slide credit: Václav Hlaváč, Bastian Leibe

Examples

[D. Comaniciu and P. Meer, Mean Shift: A Robust Approach toward Feature Space Analysis, PAMI 2002]

slide credit: Václav Hlaváč, Bastian Leibe, Svetlana Lazebnik

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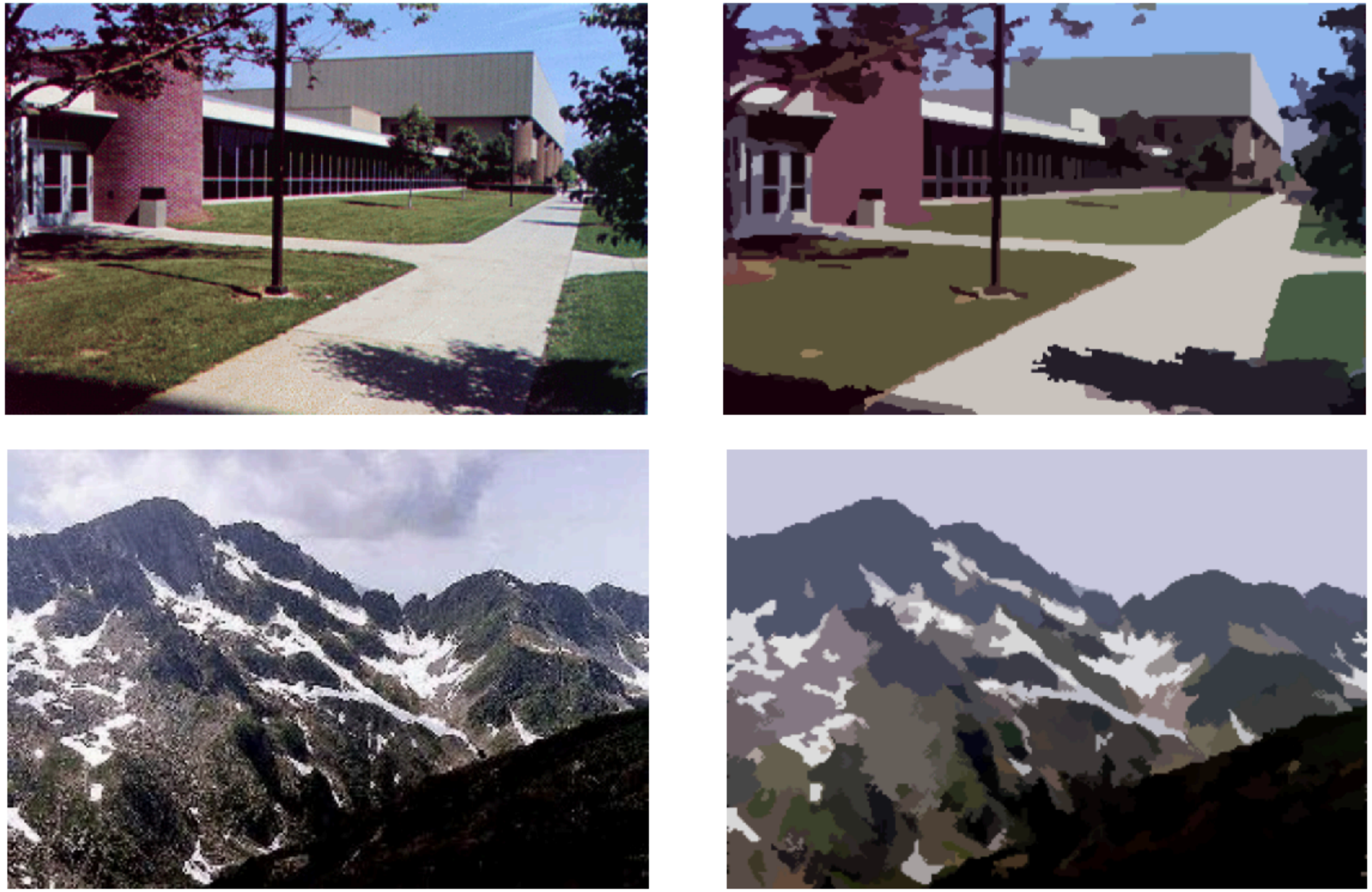

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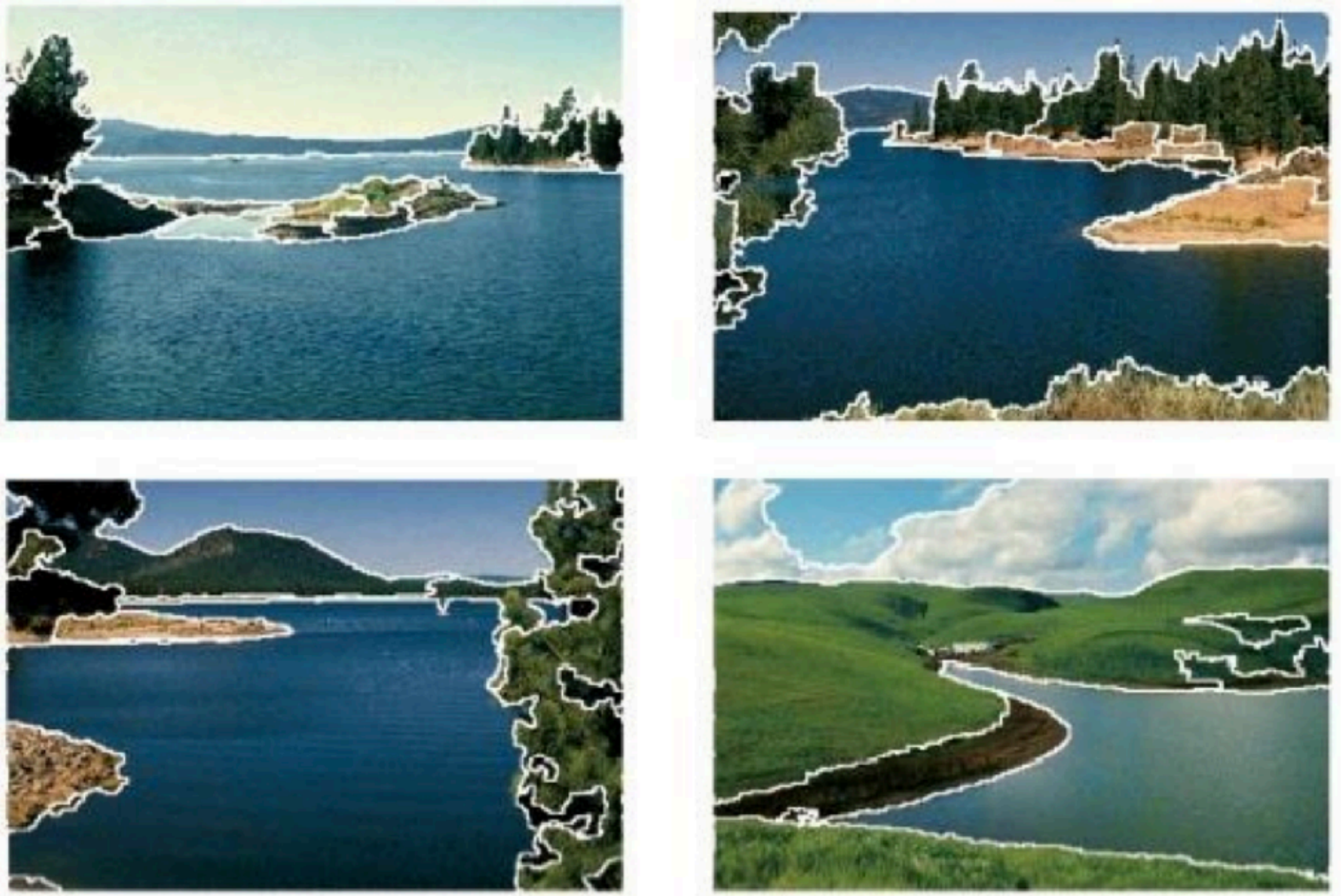

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Mean Shift Clustering - Discussion

- **Pros**:
  - Model-free, does not assume any prior shape (spherical, elliptical, etc.)
  - Single parameter with physical meaning (window size h)
  - No need to specify number of modes, finds variable number of modes
  - Robust to outliers

slide credit: Václav Hlaváč, Bastian Leibe, Svetlana Lazebnik

Mean Shift Clustering - Discussion

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  - Model-free, does not assume any prior shape (spherical, elliptical, etc.)
  - Single parameter with physical meaning (window size h)
  - No need to specify number of modes, finds variable number of modes
  - Robust to outliers
- **Cons**:
  - Sensitive to window size h
  - Selecting right window size h not trivial
  - Computationally (relatively) expensive
  - Does not scale well with respect to feature space dimension

slide credit: Václav Hlaváč, Bastian Leibe, Svetlana Lazebnik

Lecture Overview

simple & heuristic

- A simple approach to segmentation: **(intensity) thresholding**
- Segmentation based on spatial coherence: **edge-based segmentation**, **region growing**
- Segmentation as a **clustering** problem: **k-means clustering**, **mean-shift clustering**
- Segmentation as a statistical (unsupervised) learning problem: **expectation maximization (EM) algorithm**
- Next lecture: **graph-based segmentation**, **supervised learning with neural networks** (if time and interest)

complex & principled

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A Statistical Learning Perspective on Clustering

- Basic questions of practical relevance:
  - What is the shape of each cluster?
  - What is the probability a point *p* belongs to cluster *c*?

slide credit: Bastian Leibe

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A Statistical Learning Perspective on Clustering

- Basic questions of practical relevance:
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- k-means clustering cannot answer these questions

- **Statistical approach**:
  - There is a **generative model**: function relating observations $x \in X$ and their hidden state (class label) $y \in Y$
  - Described via the joint probability measure $p(x, y \mid \Theta)$ defined by parameters Θ
  - Want to **learn the parameters from data**

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Supervised Learning

- If we have $p(x, y \,|\, \Theta)$, we can define classifier / decision function $y = q(x \,|\, \Theta) = \text{argmax}_y p(x, y \,|\, \Theta)$

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x_i

y_i

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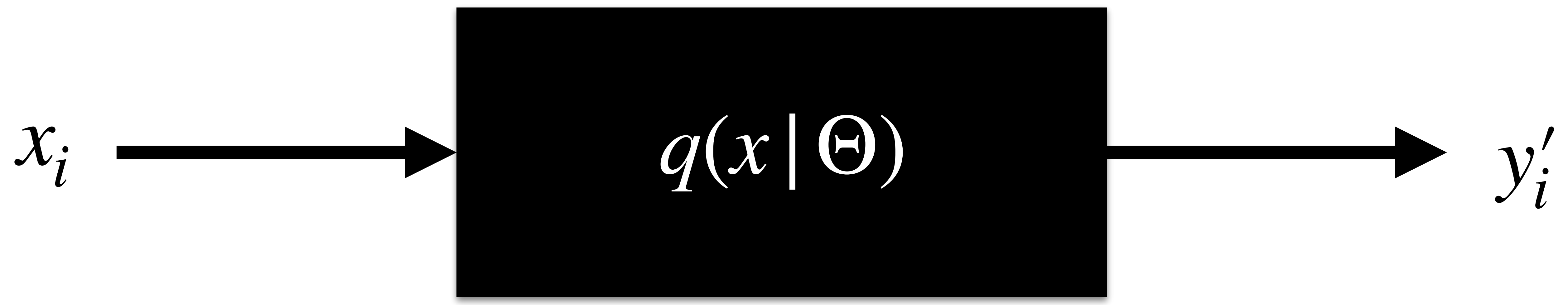

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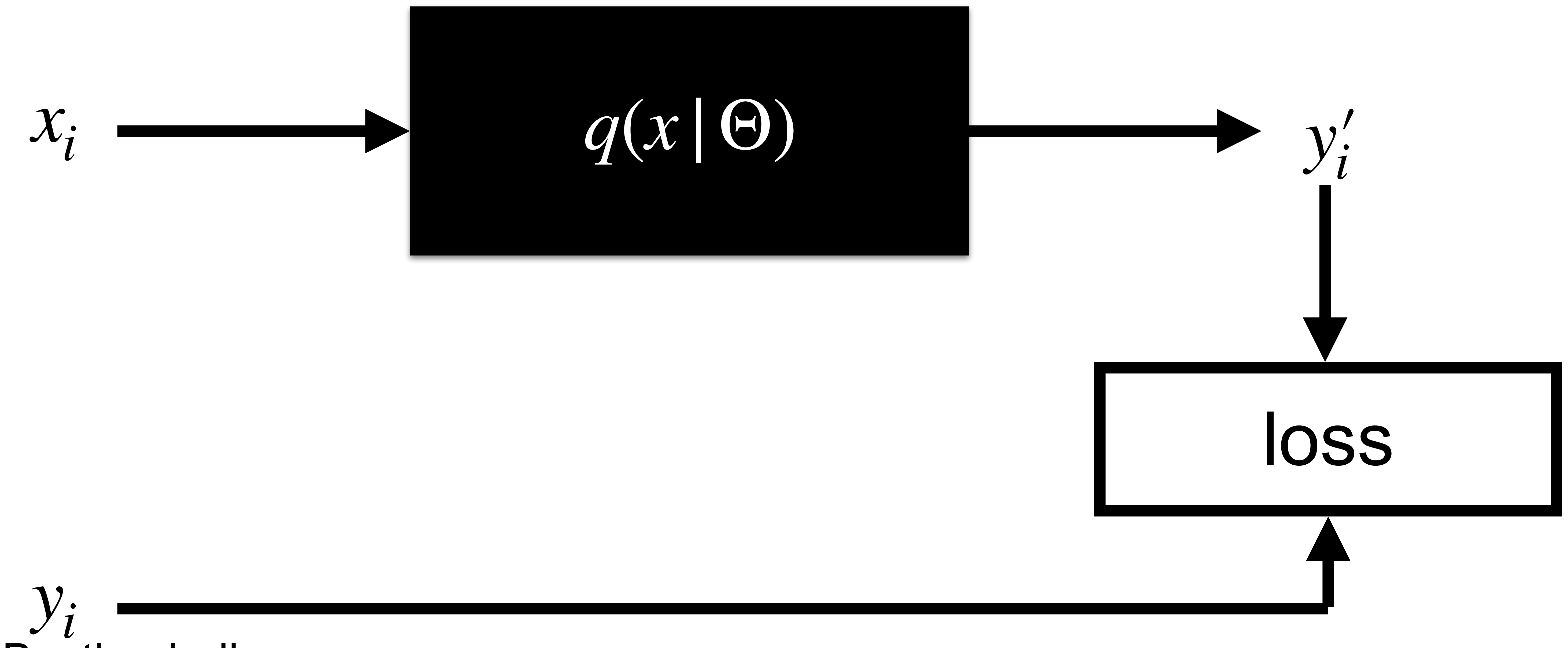

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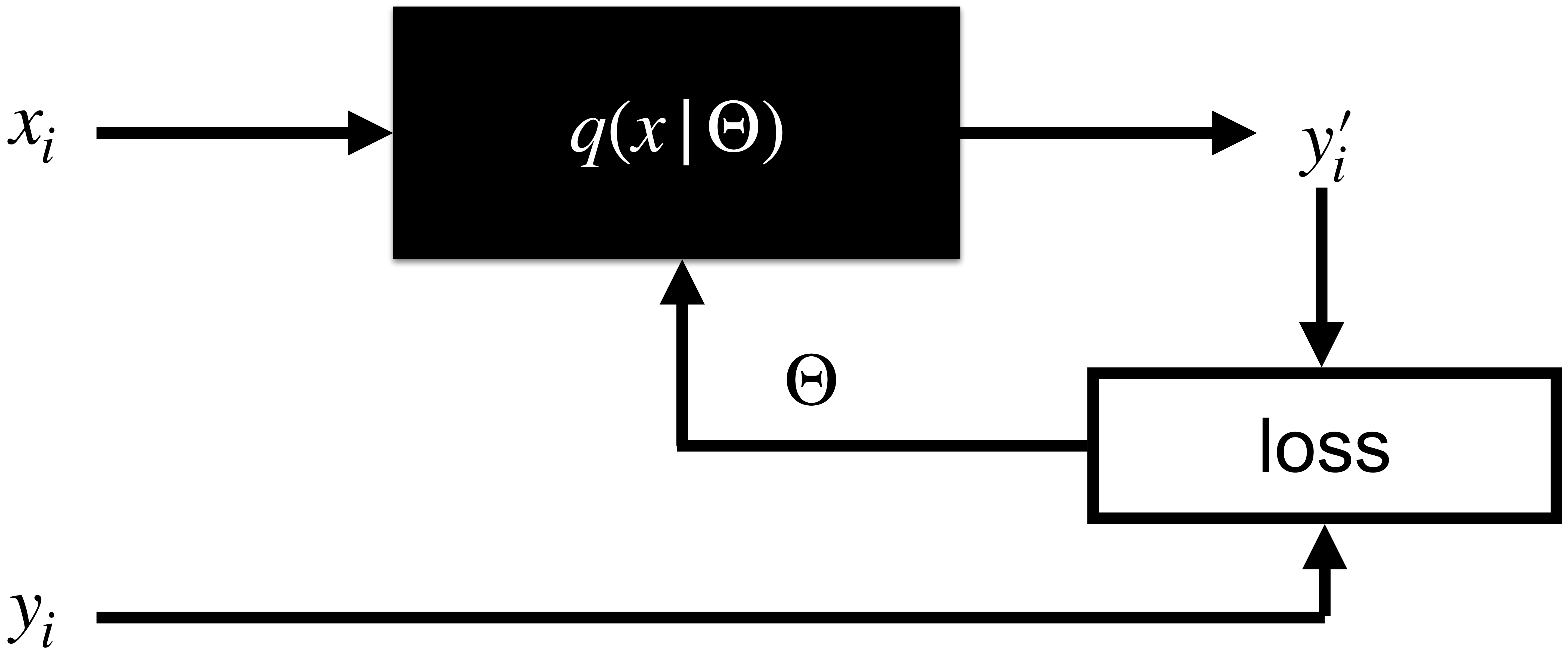

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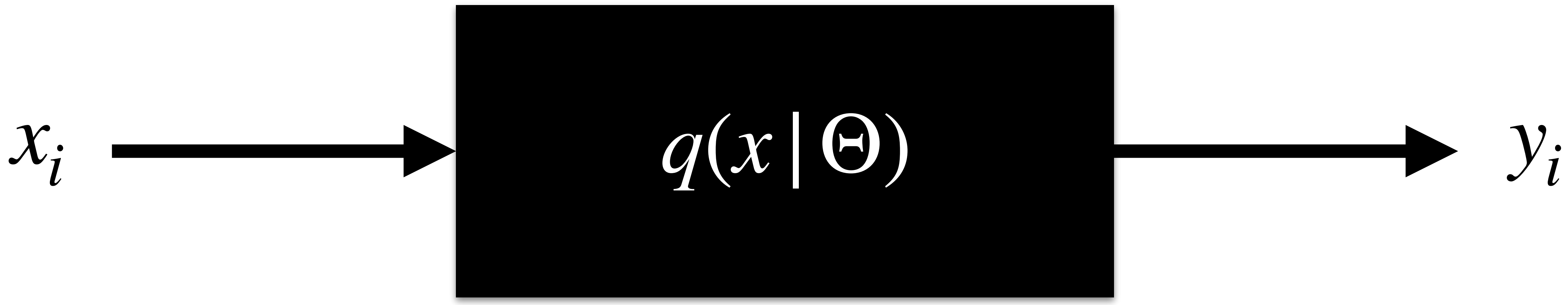

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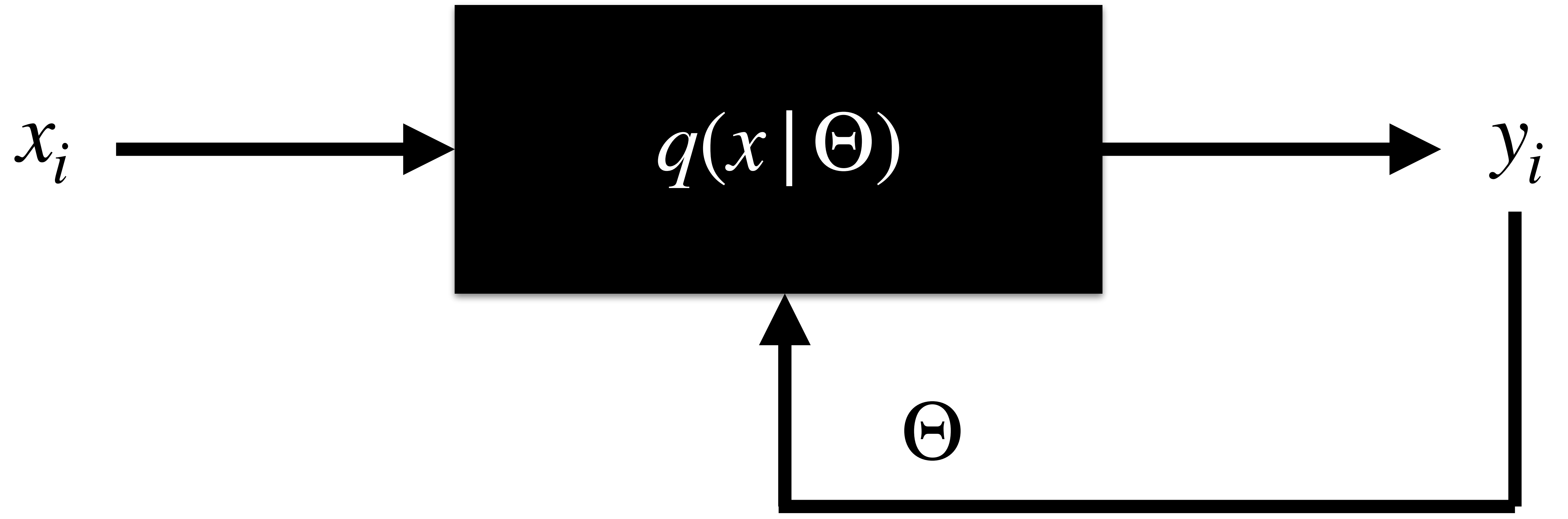

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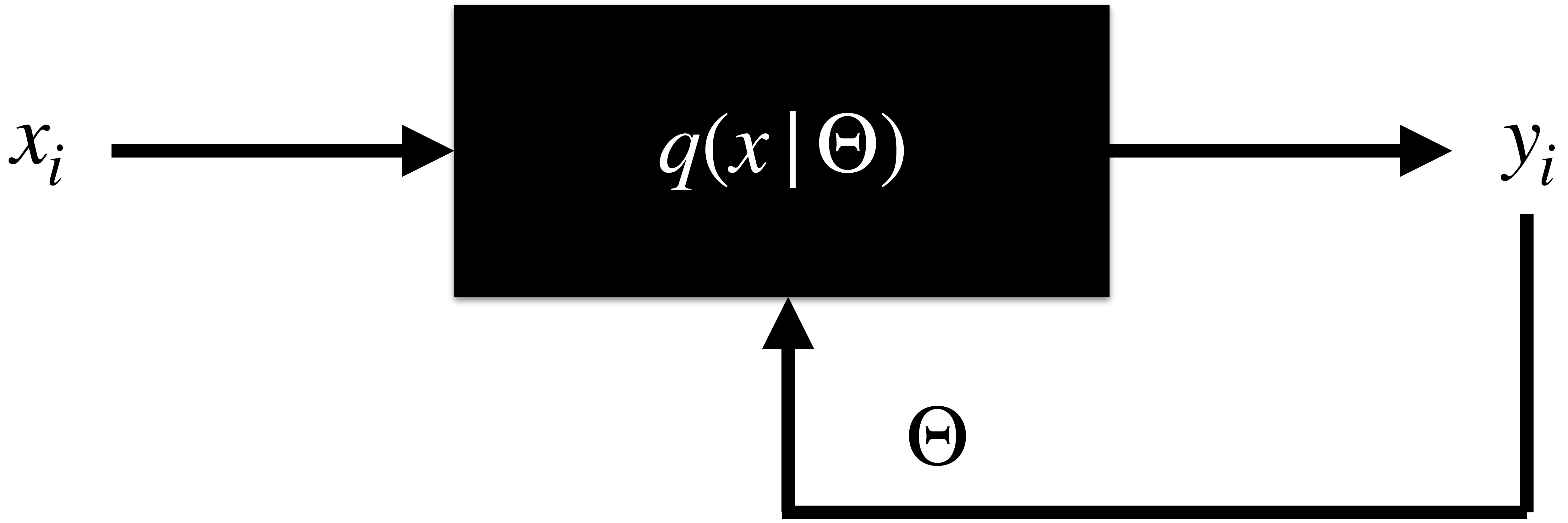

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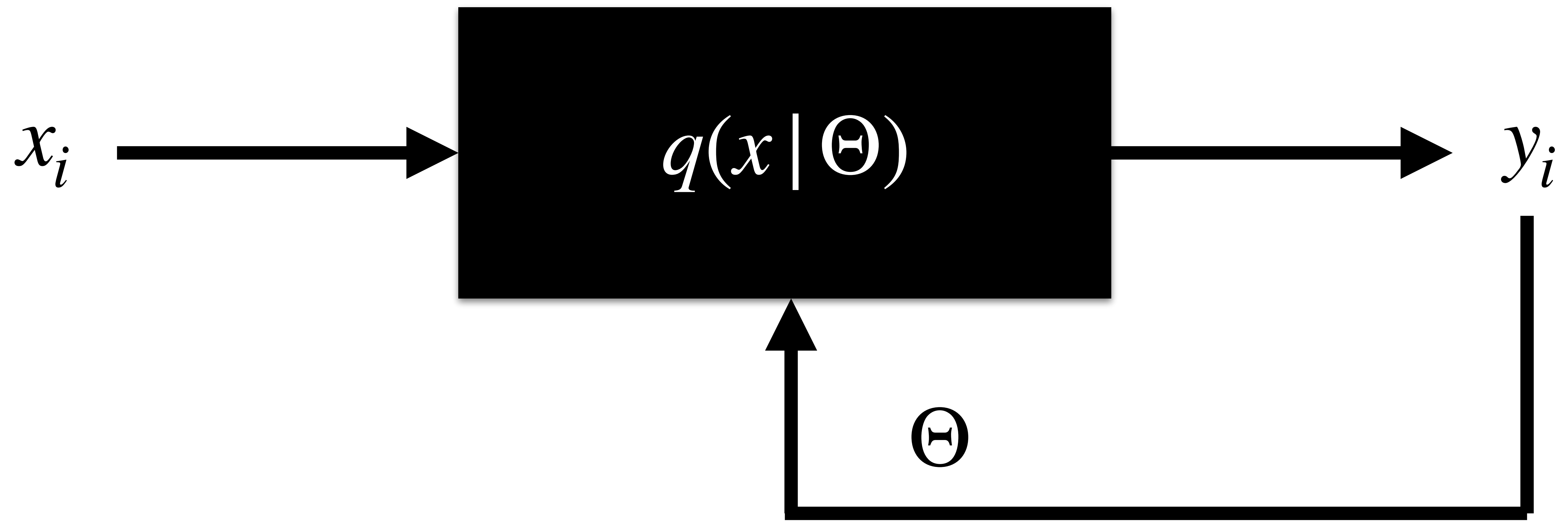

- **Chicken-and-egg problem**: if we have Θ, we can compute $y = q(x \mid \Theta)$; if we have y, we can compute Θ

slide credit: Václav Hlaváč, Bastian Leibe

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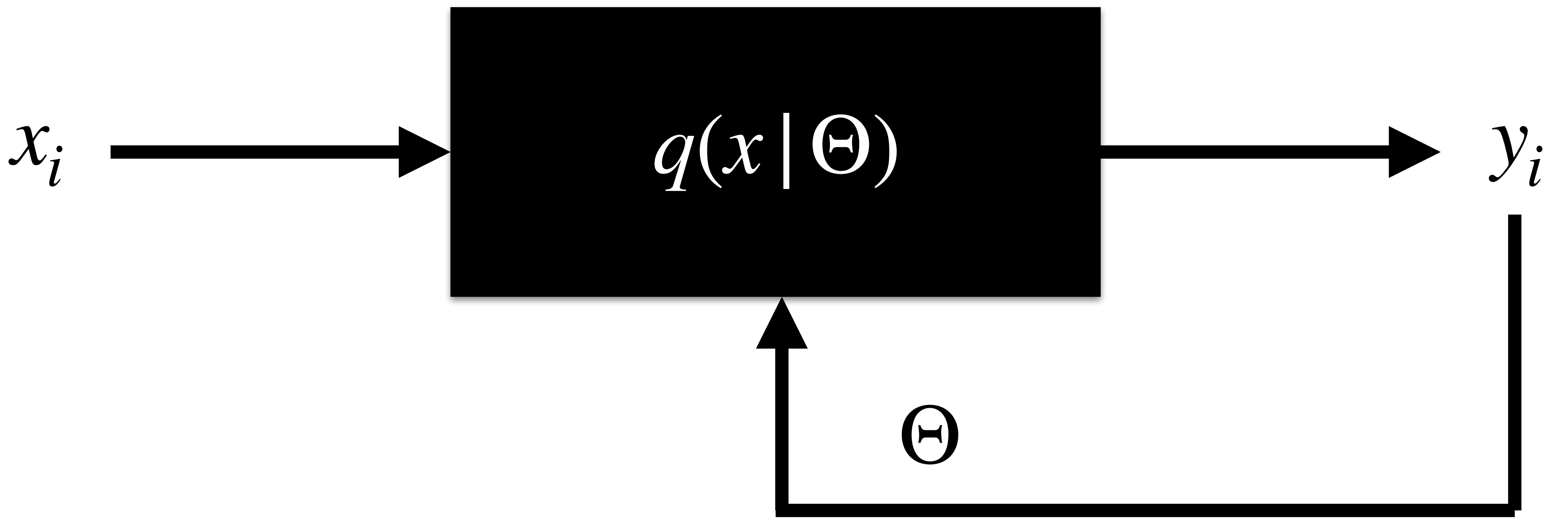
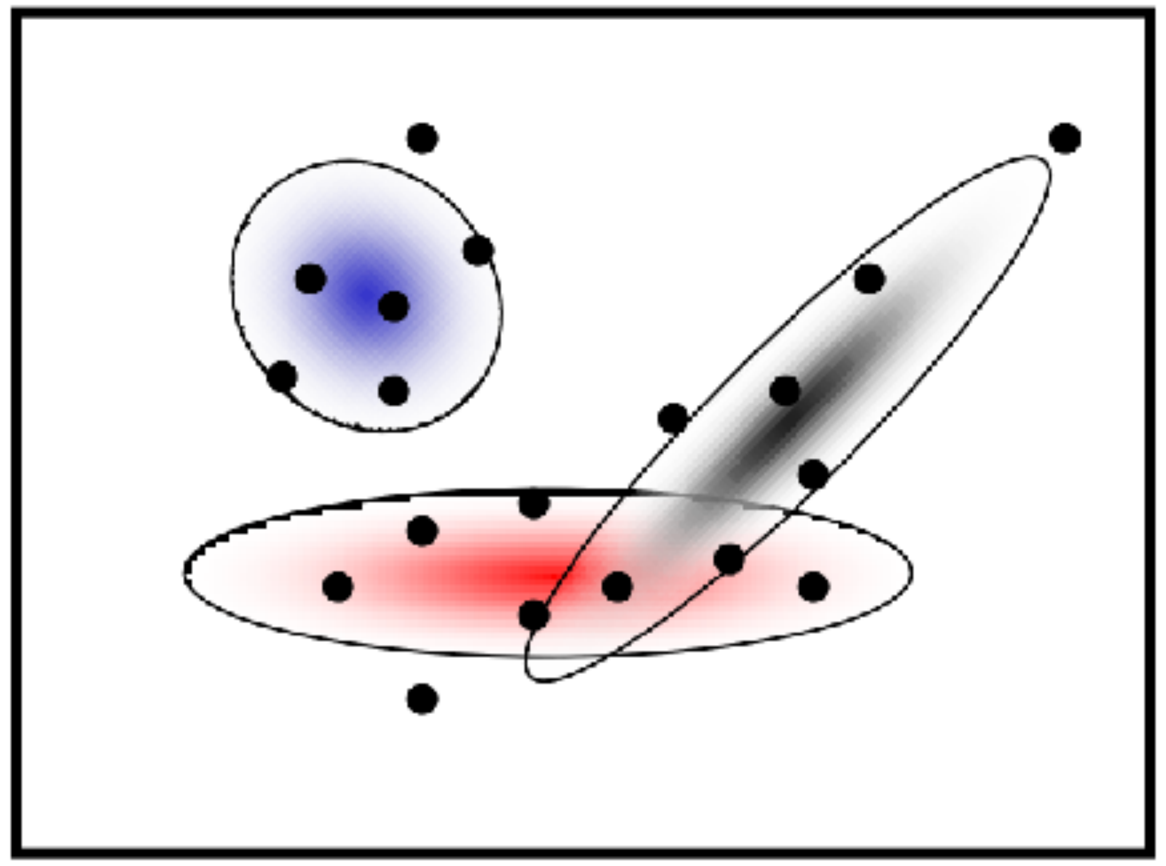


- **Chicken-and-egg problem**: if we have Θ, we can compute $y = q(x \mid \Theta)$; if we have y, we can compute Θ
- Sounds familiar?

slide credit: Václav Hlaváč, Bastian Leibe

Mixture of Gaussians



- Mixture of Gaussians is one generative model:
  - K Gaussian blobs with means μ_j, cov. matrices Σ_j, dimensionality D

$$p(x \mid \Theta_j) = \frac{1}{(2\pi)^{D/2} \mid \Sigma_j \mid^{1/2}} \exp\left(-\frac{1}{2}(x-\mu_j)^T \Sigma_j^{-1}(x-\mu_j)\right)$$

  - Gaussian j selected with probability π_j
  - Likelihood of observing data point **x** is weighted mixture of Gaussians:

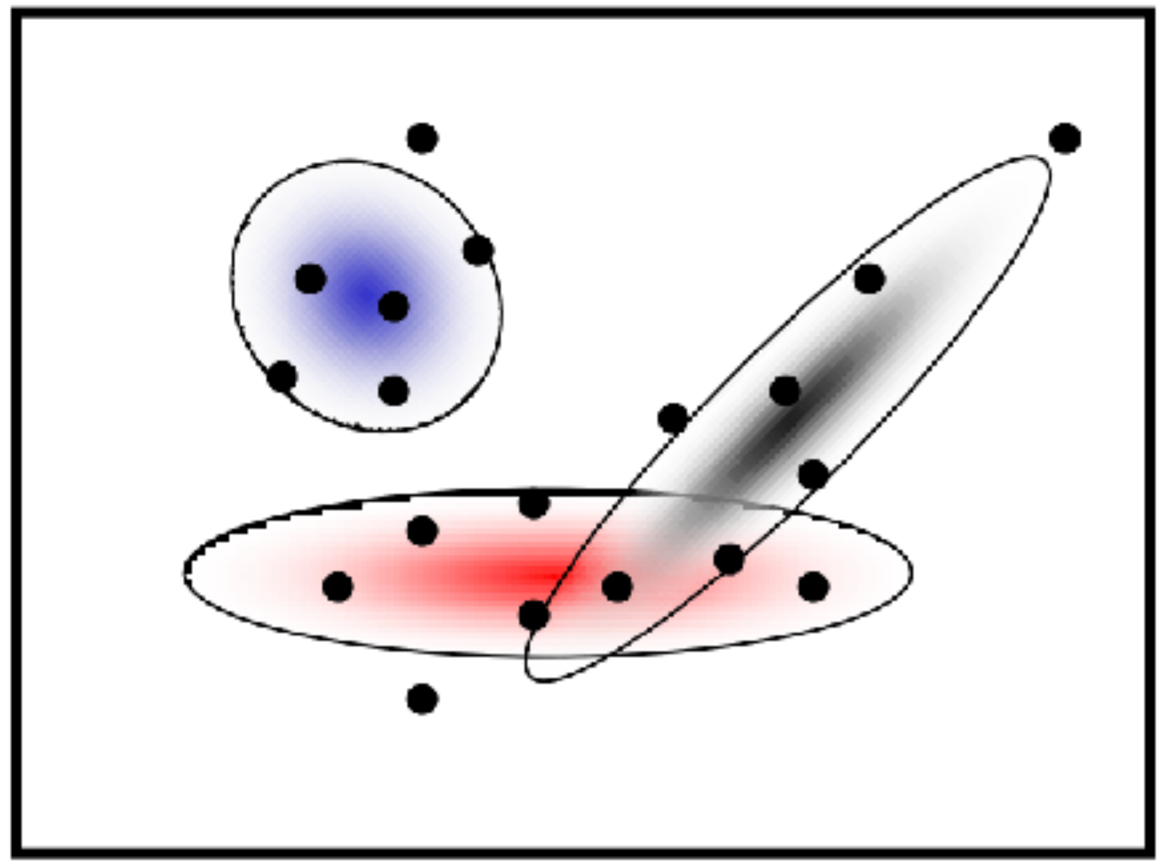
$$p(x \mid \Theta) = \sum_{j=1}^{K} \pi_j p(x \mid \Theta_j) \quad \Theta = \left(\pi_1, \mu_1, \Sigma_1, \ldots, \pi_K, \mu_K, \Sigma_K\right)$$

slide credit: Bastian Leibe

Expectation Maximization (EM) Algorithm

- Goal: find parameters Θ that maximize likelihood function:

$$p(\text{data} \mid \Theta) = \prod_{i=1}^{n} p(x_i \mid \Theta)$$



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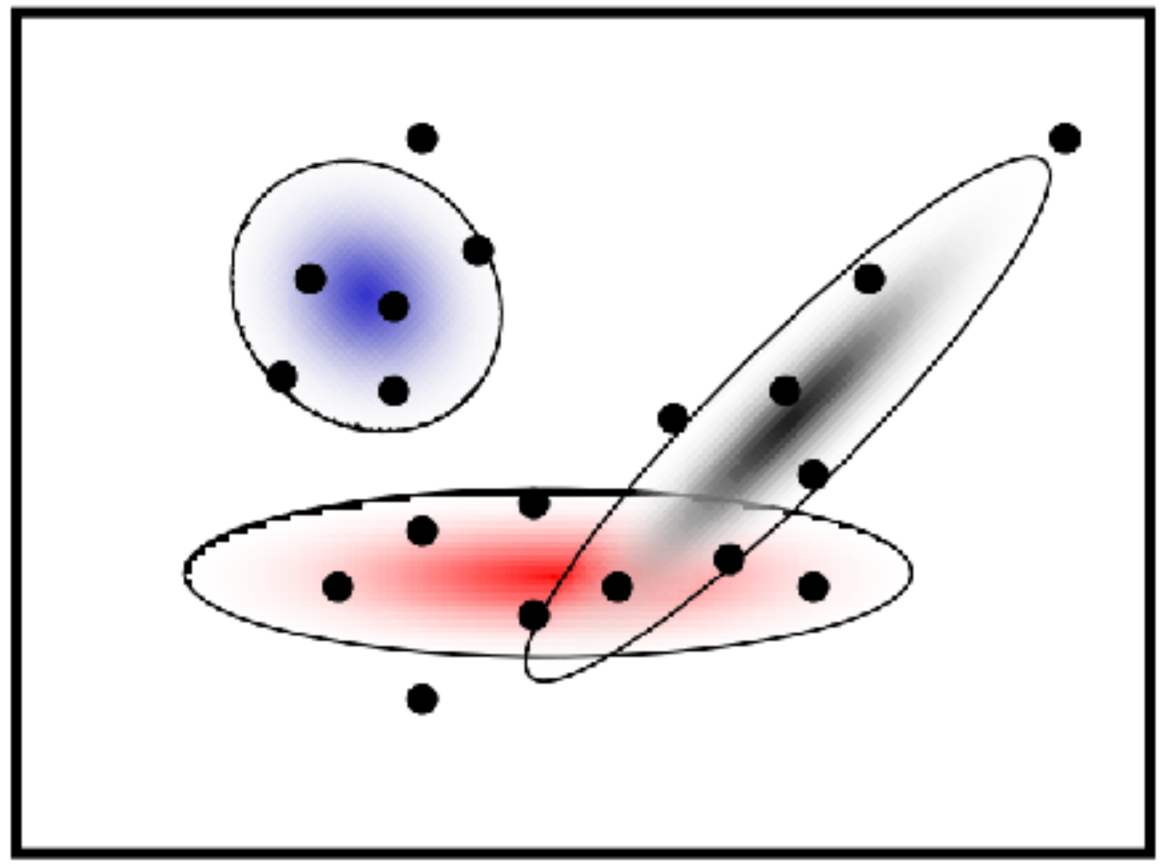

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slide credit: Bastian Leibe, Steve Seitz

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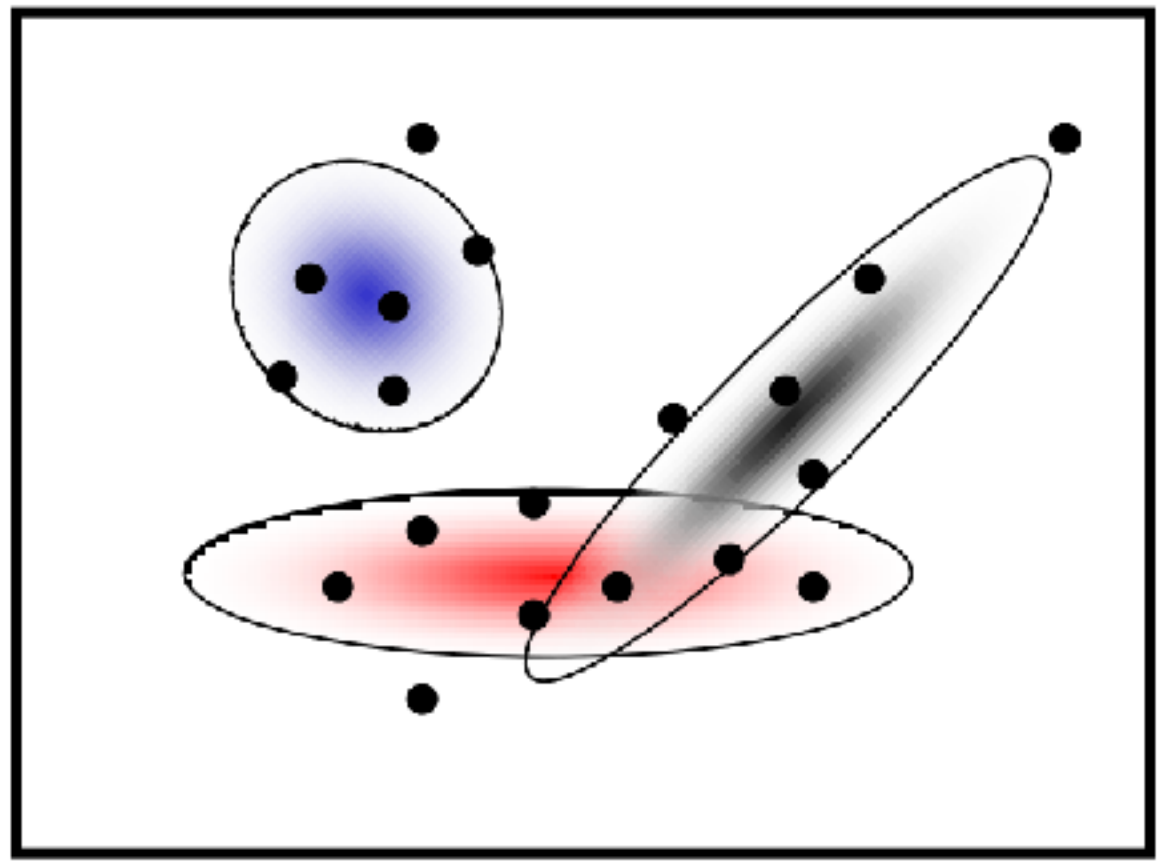

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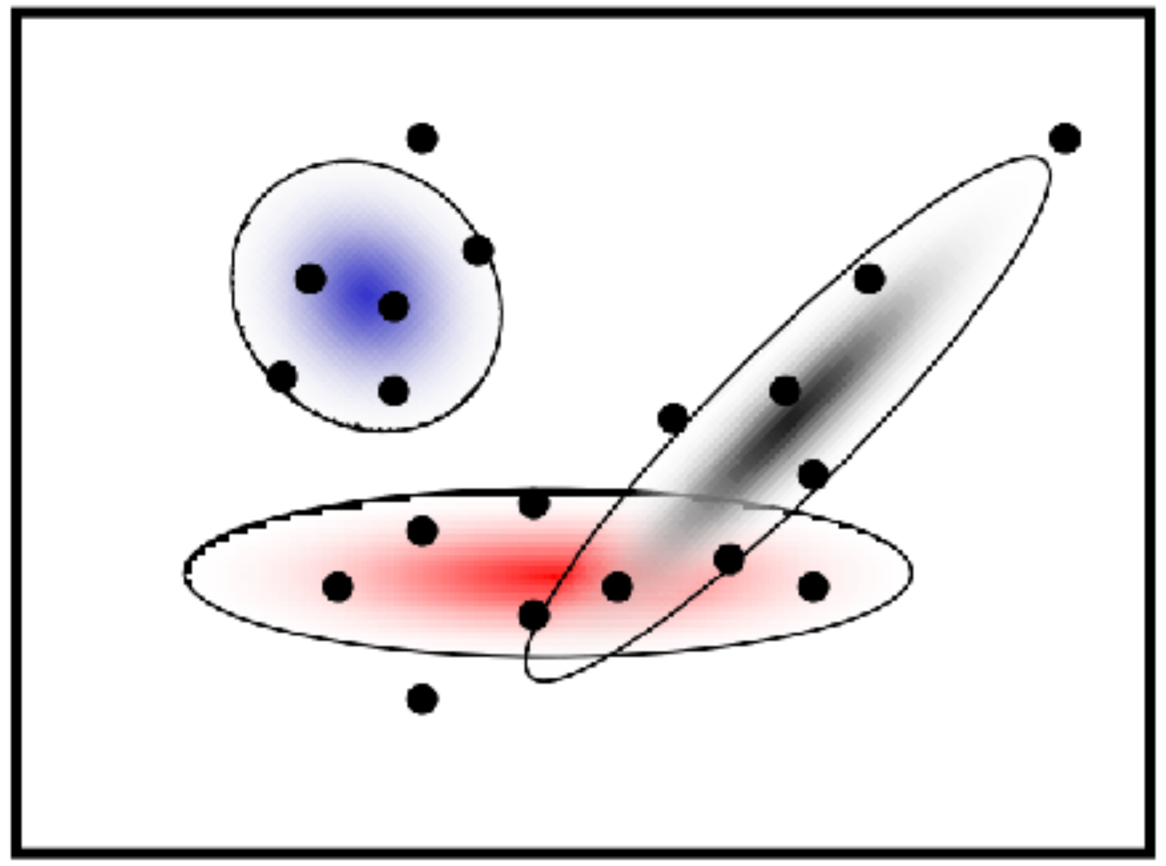

- **Expectation Maximization** (EM) approach:
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  - Repeat:

slide credit: Bastian Leibe, Steve Seitz

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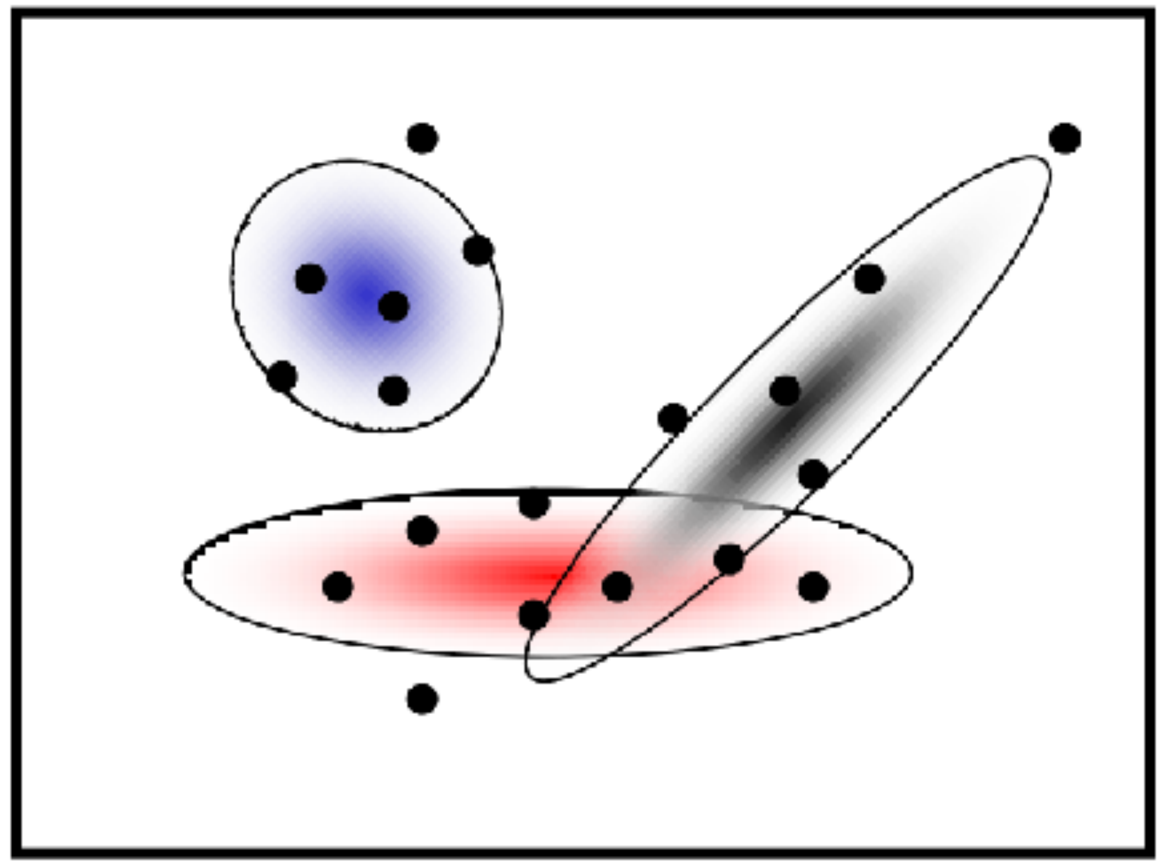

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slide credit: Bastian Leibe, Steve Seitz

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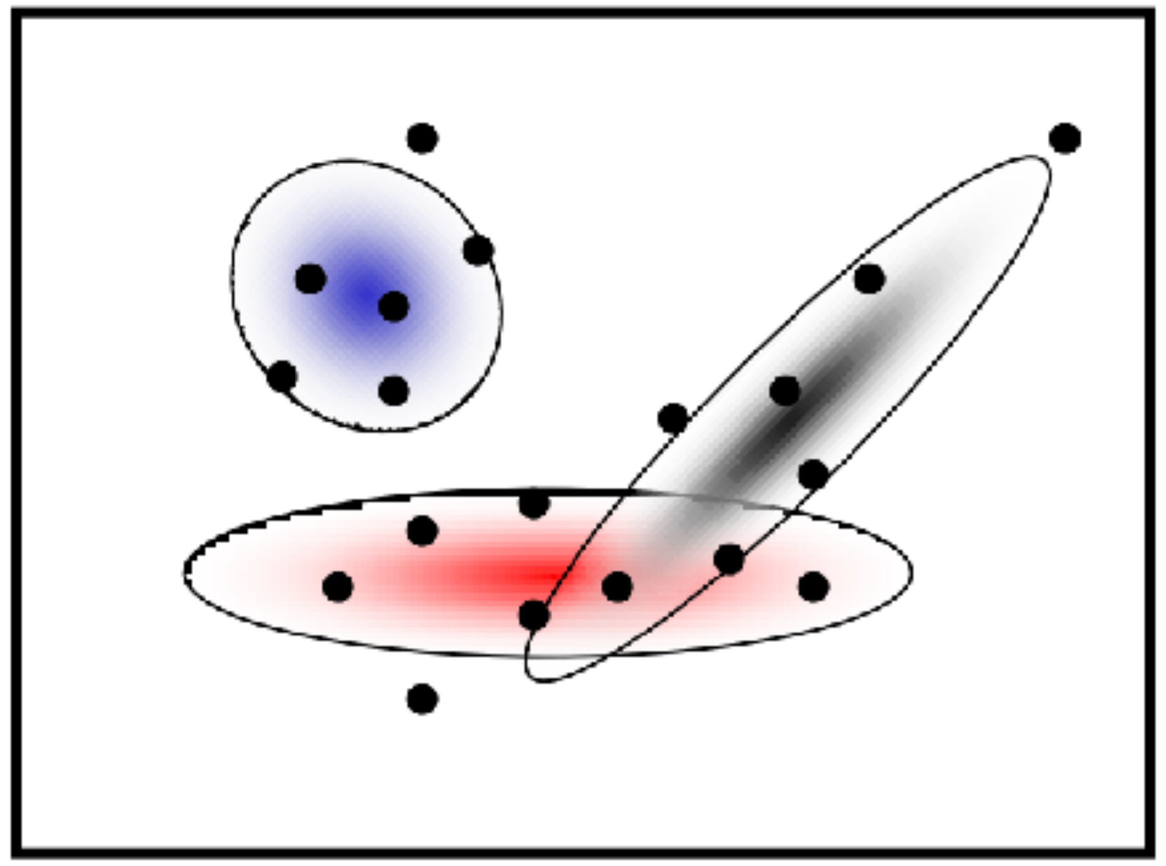

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    - **M-step**: given assignments, estimate Θ^{i+1} by maximizing likelihood function

slide credit: Bastian Leibe, Steve Seitz

Expectation Maximization (EM) Algorithm

- **E-step**: compute soft assignment of data points to mixture components:

$$\gamma_j(x_i) = \frac{\pi_j \mathcal{N}(x_i \mid \mu_j, \Sigma_j)}{\sum_{k=1}^{K} \pi_k \mathcal{N}(x_i \mid \mu_k, \Sigma_k)}$$

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$N_j = \sum_{i=1}^{n} \gamma_j(x_i)$ = soft number of points assigned to cluster j

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$$N_j = \sum_{i=1}^{n} \gamma_j(x_i) = \text{soft number of points assigned to cluster } j$$

$$\pi_j^{\text{new}} = N_j / n = \text{probability of component } j$$

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$\pi_j^{\text{new}} = N_j / n$ = probability of component j

$\mu_j^{\text{new}} = \frac{1}{N_j} \sum_{i=1}^{n} \gamma_j(x_i) x_i$ = new cluster center

slide credit: Bastian Leibe

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- **E-step**: compute soft assignment of data points to mixture components:

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- **M-step**: use soft assignments to re-estimate parameters Θ_j per component:

$$N_j = \sum_{i=1}^{n} \gamma_j(x_i)$$ = soft number of points assigned to cluster j

$$\pi_j^{\text{new}} = N_j / n$$ = probability of component j

$$\mu_j^{\text{new}} = \frac{1}{N_j} \sum_{i=1}^{n} \gamma_j(x_i) x_i$$ = new cluster center

$$\Sigma_j^{\text{new}} = \frac{1}{N_j} \sum_{i=1}^{n} \gamma_j(x_i)(x_i - \mu_j^{\text{new}})^T (x_i - \mu_j^{\text{new}})$$

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k-Means as Expectation Maximization

- **k-means clustering** is a special case of EM algorithm

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$$\Rightarrow \mu_j = \frac{1}{|\{i \mid k_i = j\}|} \sum_{\{i|k_i=j\}} x_i, \quad j = 1,\ldots,K$$

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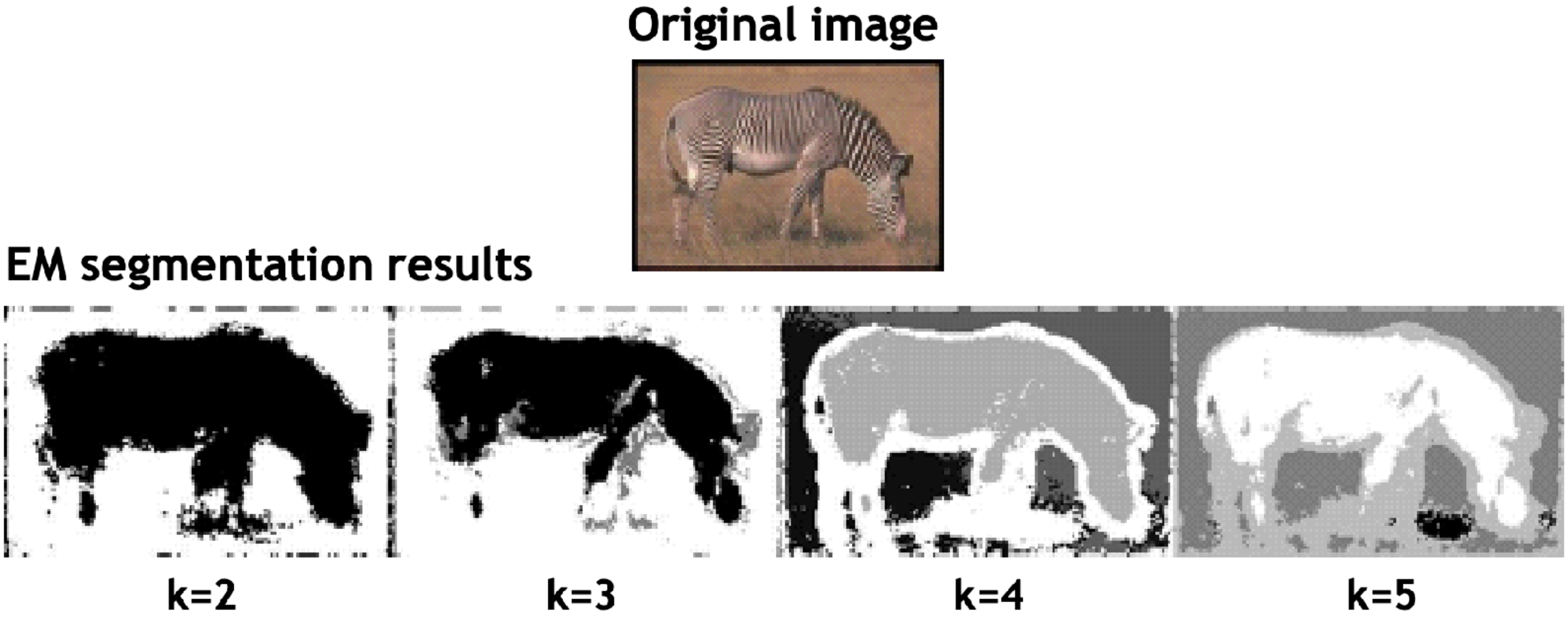
EM Algorithm for Segmentation



slide credit: Bastian Leibe

image credit: Serge Belongie

The EM Algorithm

- General statistical approach for missing data / data with hidden states

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- **No general guarantees about global optimality**, EM is essentially gradient ascent

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EM Algorithm Maximizes Lower Bound on Likelihood

- Starting from initial estimate Θ^0, iterate:

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$$L(\Theta) = p(\text{data} \mid \Theta) = \prod_i p(x_i \mid Theta) = \prod_i \sum_y p(x_i, y \mid \Theta)$$

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  - **M-step**: estimate Θ^{t+1} that maximizes lower bound

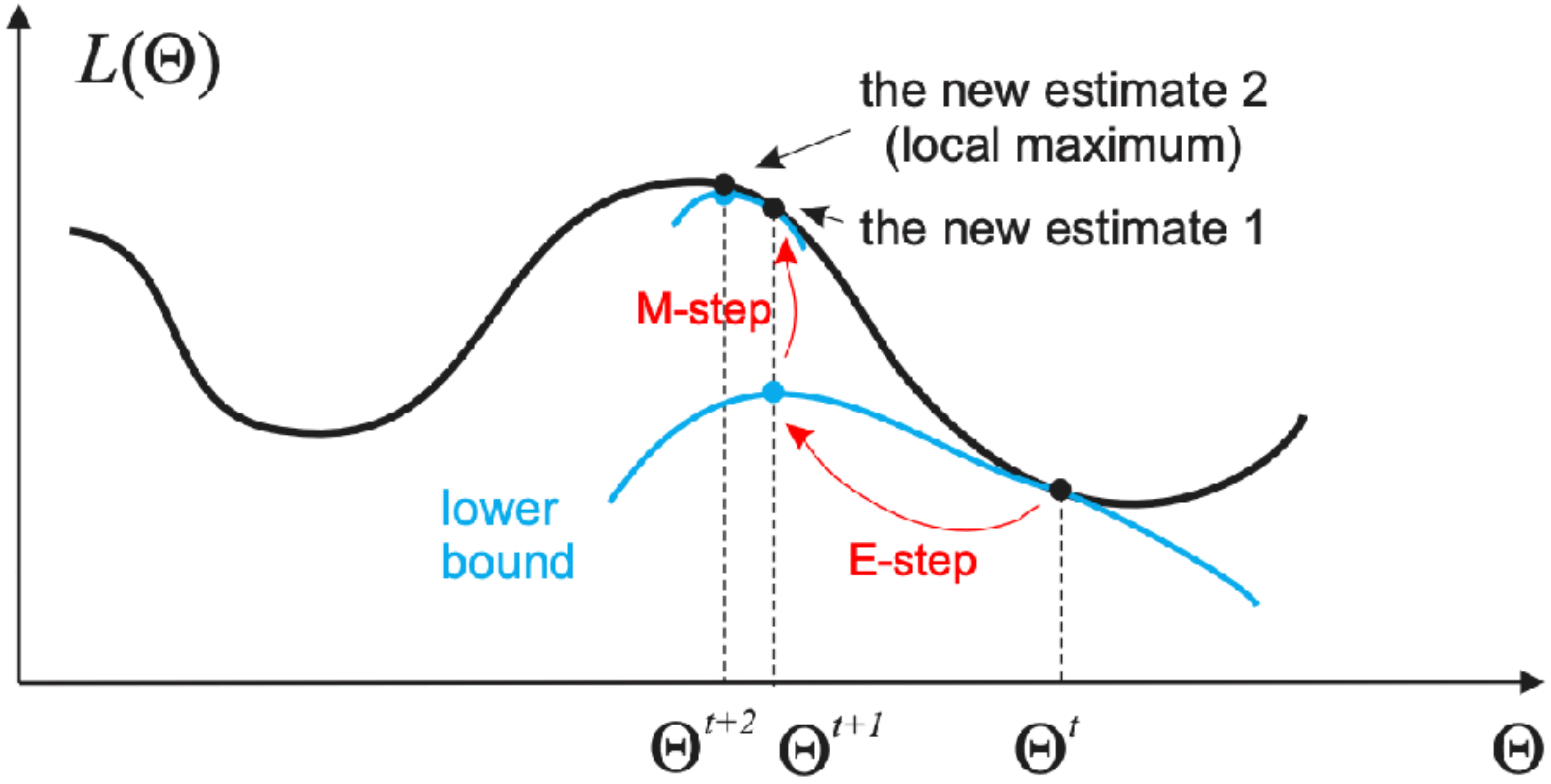

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image credit: Serge Belongie

The EM Algorithm

- **Pros**:
  - Probabilistic interpretation of data
  - Soft assignments instead of hard assignments
  - Generative model: can predict new datapoint
- **Cons**:
  - Local optimization will lead to local minima
  - Initialization is thus important (e.g., use k-means for initialization)
  - Similar to k-means clustering: need estimate for K
  - Need to choose proper generative model
  - Numerical instabilities can be an issue

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